# THE HOME PAGE OF WADE BLOCKER

Comments will be welcomed.
wblocker@nmol.com

---

This page contains links to technical papers by Wade Blocker dealing with the application of the ordinary theory of electronic feedback oscillators to acoustic feedback oscillators. It is suggested that the papers be read in the order listed. To select a paper, left click on the the colored link.

---

## The theory of the edgetone oscillator

Size of file about 140 KB

---

## A theoretical model for the edgetone oscillator

Size of file about 9 KB

---

## The Development and History

## of

## "The Theory of the Edgetone Oscillator"

Size of file about 22 KB

---

## Distributed Circuit Oscillators

### Electronic Feedback Oscillators

### and

### Acoustic Feedback Oscillators

### Compared and Contrasted

Size of file about 14 KB

---

## On the Pitch of Organ Pipes

### An Analysis of Data of Lord Rayleigh's

Size of file about 30 KB

---

## The Edgetone Oscillator as The Exciting Agent

## for

## The Organ Flue Pipe

Size of file about 14 KB

---

## A THEORETICAL PREDICTION

### OF THE

# WEISENBERGER SCALING LAWS

### FOR THE

## ORGAN FLUE PIPE

Size of file about 16 KB

---

Comments will be welcomed.
wblocker@nmol.com

# The Theory Of The Edgetone Oscillator

## Initially Written 1974; Revised 1999 and 2000

Last Revision: Oct 6, 2000; minor notes added March, 2001


## Wade Blocker

wblocker@nmol.com


## Table of Contents




Return to The Home Page of Wade Blocker.


## Abstract


A theory of the edgetone oscillator is presented which predicts all critical experimental data. This is the first theory to accomplish this. The edgetone oscillator is proved to be a transit time oscillator closely analogous to electronic oscillators. No empirical elements are required in the theory in order to fit the experimental data.



Return to the Table of Contents

Return to The Home Page of Wade Blocker.


## Introduction

Although such musical instruments as the panpipe, recorder, flute, organ flue pipe, and common whistle had been known for millennia, it was only discovered in 1854 by Sondhaus (Ref. 1) that the resonant acoustic column or cavity associated with these instruments is not necessary for a tone to be produced. A tone is

produced when a jet of fluid from an aperture is blown against an edge. Since the discovery of these edgetones an extensive literature has accumulated, but past efforts to explain this edgetone oscillator have been unsuccessful. In 1940 Lenihan and Richardson (Ref. 2) wrote *"The problem of edge tones is one which continues to form a battle-ground for rival theories, though a complete solution seems as far off as ever."* This statement has remained a challenge for theorists to this date. No prior theory of edgetones has been able to predict the data of basic experiments. In this paper it is assumed that the edgetone acoustic oscillator is just another feedback oscillator that can be explained in a manner closely analogous to how ordinary electronic feedback oscillators are explained. The assumption that the edgetone oscillator is a transit time oscillator very much like electronic transit time oscillators leads to a theory in almost exact numerical agreement with all critical experimental data, with no empirical or *ad hoc* elements introduced to force a fit to the data.

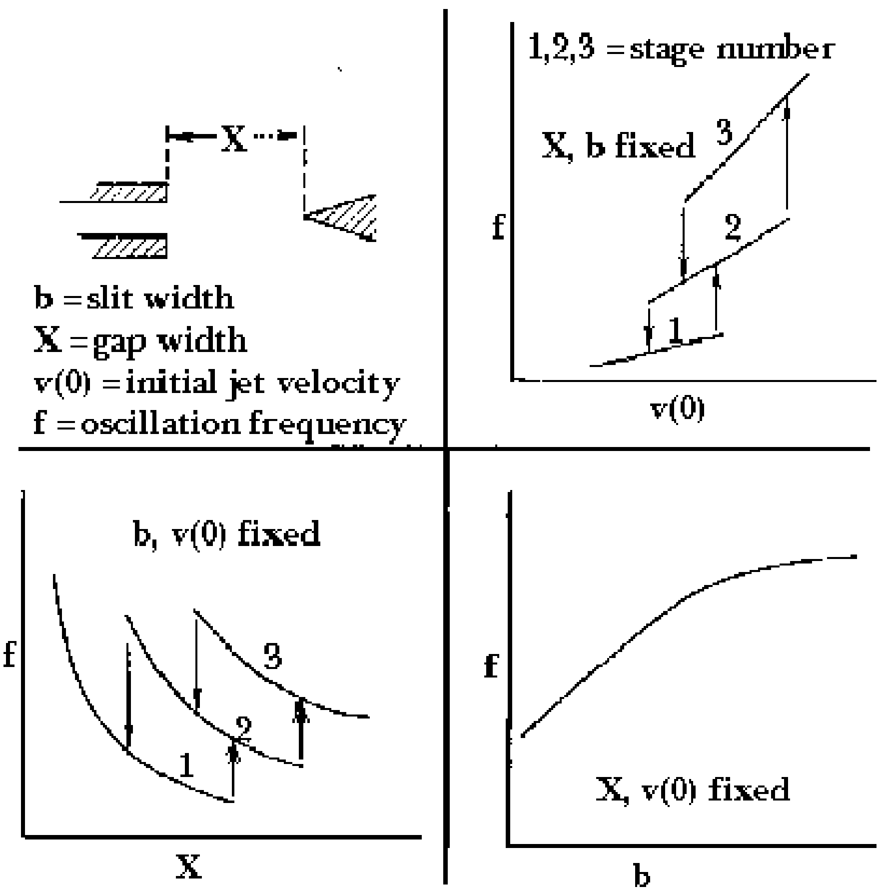
Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Edgetone Phenomenology

The basic phenomenology of edgetones seems simple. A tone is produced when a jet of fluid from an aperture is blown against an edge. Oscillations occur in both liquids and gases. Figure 1 shows the experimental facts.

# Figure 1




The upper left quadrant of Figure 1 defines the significant parameters in edgetone production. The jet and

aperture-edge system are immersed in a surrounding fluid. Denote the aperture slit width (that is, the distance between the upper and lower boundaries of the slit in Figure 1, or equivalently the thickness of the jet immediately out of the aperture) by b, the aperture to edge gap distance by X, and the velocity of the jet particles immediately out of the jet aperture by v(0). These parameters b, X, and v(0) fix the frequency f of oscillation, and it is also necessary to specify the oscillation mode or stage number. In this drawing take the origin of coordinates at the center of the aperture's exit, with the positive direction of the x-axis extending to the right through the point of the edge, the positive direction of the y-axis extending upward in the plane of the drawing perpendicular to the x-axis, and the positive direction of the z-axis extending outward perpendicular to both x and y axes toward the reader. The x-coordinate of the edge point is X. The edge and the plane jet of fluid emitted from the aperture in the positive x-direction will be considered infinite in both z-directions, thereby reducing the problem to be analyzed to two dimensions. The x-z plane divides space into two halfspaces, an upper and a lower.

The initial velocity of the jet particles immediately out of the jet aperture is v(0), and the velocity of the jet particles at the position x in the aperture to edge gap is v(x). At the edge position X the jet particles have slowed to the velocity v(X). The failure to give attention to the slowing of the jet particles is almost surely the major reason for the previous failures to develop an adequate theory of the edgetone oscillator.

The upper right quadrant of Figure 1 shows the effects of changing v(0) for fixed b and X. As the velocity increases from zero, oscillations begin at some point and the edgetone is produced. The oscillation frequency then increases with velocity along the lowest line indicated by the numeral 1. At some velocity the frequency jumps upward and then increases along line 2. With a further velocity increase the frequency jumps to line 3. As many as five or six jumps have been seen. Beginning with the lowest line, these modes of oscillation are called stage 1, stage 2, stage 3, etc. With a velocity decrease the frequency decreases along one of the lines and downward jumps back to stage 1 occur. The upward and downward jumps do not necessarily occur at the same points so this oscillator shows hysteresis just as most oscillators do. The lines are straight, and if extended go through or very near the origin.

The lower left quadrant of Figure 1 shows the effect of changing X for fixed b and v(0). As the gap width from aperture to edge increases, oscillations begin at some point. The frequency then drops as the gap width increases. There are frequency jumps as the gap is increased and again hysteresis effects are seen. The stages identified are the same as those seen when the velocity was varied. Most experimenters have found the frequency in a fixed stage to vary inversely as the first power of the gap width, but some experiments have been done in which the frequency varied inversely as the three-halves power of the gap width. These latter experiments have been mostly ignored by theorists attempting to explain edgetones, which is another reason for previous failures to explain edgetones.

The lower right quadrant of Figure 1 shows the effect of changing the slit width b for fixed v(0) and X. As b increases, oscillations begin at some point. The frequency then increases as the slit width increases and appears to approach a limit. Presumably jumps between stages might occur but they have not been reported. Most papers on edgetones do not identify the slit width b as an important parameter of these oscillators but give attention only to initial jet particle velocity v(0) and aperture to edge gap width X. But the slit width b is as important as the gap width X in determining the edgetone frequency.

Most writers on edgetones would add to the experimental facts just given that the jet disturbance in the gap when oscillations are occurring is characterized by a phase velocity which is about half the jet particle velocity, both velocities being tacitly assumed constant across the aperture to edge gap. This is an inference from certain theoretical and experimental work. The present paper shows that this inference is incorrect. Its general acceptance by theorists is another major reason for prior failures to develop an adequate theory of edgetone production.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Past Experiments and Theories

The number of experimental and theoretical papers written on edgetones is very large, certainly in the high hundreds. Only a few of the most important will be reviewed briefly here.

For the first oscillation stage, Koenig in 1912 (Ref. 3) proposed the empirical equation

$$f = v(0)/2X \tag{1}$$

He did not explain why the factor 2 in the denominator was necessary to fit the experimental data.

Schmidtke in 1919 (Ref. 4) thought that the different frequencies observed in different stages should be harmonically related. Later more accurate experiments showed this to be incorrect. Schmidtke extended Koenig's equation as

$$f = nv(0)/2X \tag{2}$$

where n was the stage number.

Krueger in 1920 (Ref. 5) proposed that the factor 2 in the denominator of equations 1 and 2 was properly associated with the jet particle velocity v(0) rather than with the aperture to edge distance X and indicated that the phase velocity of the jet disturbance in the aperture to edge gap was only half the jet particle velocity. This proposal was later almost universally accepted by those attempting to explain edgetones. It did give a plausible reason for the otherwise unexplained factor 2. However, the proposal will turn out to be wrong. His form of the frequency equation would be

$$f = n[v(0)/2]/X \tag{3}$$

CarriÃ¨re in 1925 (Ref. 6) published experimental results showing the variation of the edgetone frequency with slit width b, with other parameters fixed. This is the only data the author is familiar with that gives frequency as a function of slit width b and is the data on which the diagram of the lower right quadrant of Figure 1 is based. CarriÃ¨re's data show that the aperture slit width b is as important as the parameters X and v(0) in determining the edgetone frequency. Nevertheless, the influence of the parameter b on edgetone frequency f has been universally neglected by edgetone theorists. The parameter b does not appear in any equation they propose for the edgetone frequency.

CarriÃ¨re also gave experimental results in which the frequency varied inversely as the three-halves power of the aperture to edge distance X. Both major experimental findings just cited of CarriÃ¨re's on edgetone phenomenology have been verified by the later experimental findings of other writers on edgetones but the findings are still mostly ignored by theorists. However, his data will be extremely important in verifying the theory to be offered in this paper.

Brown in 1937 (Ref. 7) published an extensive and perhaps the best yet collection of experimental data. His given experimental data are excellent, and are of primary importance in verifying the theory of the present paper. Brown had made frequency measurements for more than one value of slit width b and states that the edgetone frequency decreases as the slit width decreases, but Brown gives detailed data in this paper for only one slit width. This is a major deficiency in Brown's paper, not necessarily a criticism of Brown since Brown makes no claim to have covered all aspects of edgetone phenomenology, but later writers on edgetones seem to have had the opinion that Brown's paper does cover every important aspect of edgetone phenomenology. It is a fact that, except for CarriÃ¨re, and an off-hand mention by Brown that slit width

affects the edgetone frequency, all writers on edgetones have failed to give serious consideration to the possible importance of the slit width b in determining the edgetone frequency.

Brown established that the frequencies in different stages are not harmonically related. His data give the edgetone frequency for a single slit width b as a function of initial jet velocity v(0) and gap width X. Although not stated by Brown, as pointed out later in this paper it can be deduced from Brown's data that for some conditions the edgetone frequency is inversely proportional to the three-halves power of the gap width X, thus Brown's data confirm this finding of CarriÃ¨re's. Brown's purely empirical equation for the frequency, slightly simplified, is

$$f = a\, v(0) / 2X \quad (4)$$

where a = 1.0, 2.3, 3.8, 5.4 for stages 1, 2, 3, and 4.

Brown took photographs of smoke filled jets on which he was able to measure wavelengths of the jet wave near the edge. He was able to measure wavelengths only near the edge since only just before the edge was there enough detail in his photographs to allow a wavelength to be defined. These wavelengths multiplied by the frequency gave a phase velocity which was about one-half of the jet particle velocity at the aperture. Brown interpreted this to mean that the phase velocity of the jet wave was about one-half the jet particle velocity in general agreement with the supposition of Krueger. Implicit in this interpretation by Brown of his data are unstated assumptions which are incorrect. He tacitly assumed that the jet particles did not slow down, and that the wavelength and phase velocity which he determined just before the edge were the wavelength and phase velocity at all points in the gap. He gave values for the different stages of the gap width divided by his measured wavelengths, assuming this to be the number of waves in the oscillation of the jet in the gap. This number turned out to be approximately equal to or somewhat larger than the stage number, leading to the conclusion that the number of wavelengths in the gap was equal to or greater than the stage number. All of Brown's assumptions just mentioned are wrong and every conclusion of Brown's based on these assumptions is wrong. This will be discussed in detail later. Most later theorists attempting to explain edgetones relied heavily on Brown's data and his interpretation of that data. They overlooked the errors of interpretation which Brown made which this paper points out. In checking a theory against Brown's data, the excellent original data as recorded by Brown should be used and not the data as interpreted by Brown.

Brown's basic data are acknowledged to be excellent. But there are five major errors in Brown's interpretation of his data. (1) Implicit in Brown's interpretation of his data is the unstated assumption that the jet particle velocity at all positions in the aperture to edge gap is the same as the jet particle velocity just out of the aperture; in other words, Brown neglected the slowing of the jet particles as they crossed the aperture to edge gap. (2) Brown tacitly assumed (again an unstated assumption) that the wavelength in the jet disturbance in the aperture to edge gap at all positions in the gap was the same as the wavelength measured immediately adjacent to the edge. These two unstated and erroneous assumptions immediately led to three other errors. (3) Multiplying the wavelength measured just adjacent to the edge by the edgetone frequency, Brown obtained a phase velocity which he implicitly assumed to be constant and to be the phase velocity at all positions in the aperture to edge gap. (4) Taking the ratio of this phase velocity determined immediately adjacent to the edge to the jet particle velocity immediately out of the aperture, Brown concluded that the phase velocity of the jet wave at any position in the gap was a small and constant fraction of the jet particle velocity at that position in the gap, since he had already tacitly assumed that both velocities were constant and independent of position in the gap. If Brown had compared the phase velocity he measured at the edge to the actual jet particle velocity at the edge, not the jet particle velocity at the aperture, he would have found the two velocities to be for practical purposes equal to each other. (5) Since there was not enough detail in Brown's photographs of the jet wave in the gap to allow the number of wavelengths in the jet wave in the gap to be counted, Brown assumed the number of wavelengths was just the gap width divided by the wavelength he measured immediately adjacent to the edge. This resulted in Brown concluding that there were more wavelengths in the jet wave in the gap than was actually the case, since the wavelength becomes

smaller as the edge is approached since the jet particles are slowing down. All of these errors will be discussed in detail in later sections of this paper. Unfortunately not just Brown's experimental data but also Brown's interpretation of that data have been accepted by later theorists and made the basis of their attempts to explain edgetones. This is particularly true of the many papers (to be referenced later) of Powell on edgetones which will be discussed in some detail later. Particular attention is given to Powell's papers since Powell has been a very prolific writer on edgetones and his influence has been very great on many other writers who have adopted Powell's approach in their own attempts to explain edgetones. Unfortunately, as pointed out in the history of the present paper presented in another paper on this same web site, Dr. Powell has been a bar to the presentation in the published literature of this paper giving a contrary view of the interpretation of Brown's data, although Dr. Powell was aware that the contrary view enabled the prediction of every experimental fact about the edgetone oscillator that either Brown or Carriere had published.

The papers by CarriÃ¨re and Brown are presently the definitive papers that together best set forth the experimental facts of edgetone oscillations that are illustrated in our Figure 1. These are the authors whose data every theorist on edgetones should try to explain and predict. Brown's paper has received a great deal of theoretical attention, but no prior paper has presented a theory that can predict or explain Brown's experimental data. Also, the present author is not aware of any prior theoretical paper that gives attention to explaining CarriÃ¨re's results, although his results have not been challenged. CarriÃ¨re's data must be considered in order to have a complete picture of basic edgetone oscillator phenomenology.

Jones in 1943 (Ref. 8) published remarks that confirmed CarriÃ¨re's finding that for some conditions the frequency varied inversely as the three-halves power of the aperture to edge distance. Jones noticed that this occurred when the gap was wide and the jet was turbulent. Jones suggested a mechanism for the tone production similar to that adopted in the present paper. However Jones did not go beyond suggesting this mechanism to analyze quantitatively the consequences and did not produce a successful theory. Jones' remarks received little attention, although they offered the key for the solution of the edgetone problem.

Curle (Ref. 9) and Powell (Ref. 10) in 1953 both relied on the presentation and interpretation Brown gave of his data and, accepting Brown's conclusions about the number of wavelengths in the jet wave in the aperture to edge gap, each independently proposed for the edgetone oscillator frequency the purely empirical equation

$$f = [n + (1/4)]v(0)/2X \qquad (5)$$

where n is the stage number. This equation manages to conform to every erroneous feature of Brown's interpretation of his data pointed out above. It will be shown later in detail that Brown's experimental data, properly interpreted, offer no support for this equation, which must be recognized therefore as an unsupported conjecture. It cannot be used to predict the data of CarriÃ¨re and Brown. Nevertheless, Powell's paper has had a very great influence on later theorists, many of whom have taken Powell's paper and equation as a starting point in their own attempts to explain edgetones (Note added in 1999: for a recent example, see Ref. 20).

The frequency equations proposed were empirical. These equations do not give any attention to the inverse three-halves power dependency of frequency on gap width X seen by both CarriÃ¨re and Jones (and also deducible from Brown's data); nor to the strong dependency of frequency on aperture slit width b known to exist from the work of CarriÃ¨re, and confirmed by Brown although Brown gives no detailed data. The parameter b, the aperture slit width, is not a parameter of these equations; in effect, every theorist has ignored the dependence of edgetone frequency upon aperture slit width without offering any explanation for that ignoration. Theorists continued to treat the jet velocity as constant, despite the known fact that these jets slow down rapidly.

For fixed values of v(0) and X, these equations all predict that the ratio of the frequencies seen in two different stages is a function of the two stage numbers only, and is independent of the values of v(0) and X.

However there is no agreement between these equations as to the values of the ratios. These equations give no attention to how the parameter b might affect these frequency ratios. The true statement will turn out to be: "For fixed values of b, v(0), and X, the ratio of the frequencies seen in two different stages is a function of the two stage numbers only, and is independent of the values of b, v(0), and X".

Return to the Table of Contents

Return to The Home Page of Wade Blocker

## A Transit Time Oscillator, fT = k

There is another way of interpreting equation 1 than Krueger's. If X/v(0) is taken as an approximation to the time T required for the jet particles to go from the jet aperture to the edge, then equation 1 becomes fT = 0.50, so we see that it has taken one half-period of the oscillation for the jet particles to reach the edge. This suggests that a necessary phase condition on the oscillation of the jet wave in the gap is being satisfied in order that oscillations may occur. This suggests further that the edgetone oscillator may be a transit-time oscillator whose frequency is given by the equation fT = k where each oscillation stage or mode has its own value of k. The transit-time oscillator is a well known and quite common electronic oscillator.

Approximating the transit time T from aperture to edge by X/v(0) is valid for those cases in which the aperture to edge gap width X is not appreciably greater than the aperture slit width b, but we should not expect this to be a good approximation in those cases where the gap width X is very large compared to the slit width b. We should certainly not expect this to be a good approximation in those cases where the jet particles have lost a large fraction of their velocity by the time they have reached the edge distance X. It has already been stated that the jet particles slow down very rapidly in the aperture to edge gap, a fact that will be substantiated in detail later in this paper. When this slowing is taken into account properly, it will be easily demonstrated that the edgetone oscillator is in fact a simple transit-time oscillator governed by the same equation fT = k that applies to electronic transit-time oscillators.

To check the assumption that the edgetone oscillator is a transit time oscillator is simple, but since the present author has not been able to find any indications of such a check in prior published work on edgetones, it seems not to have been done previously. The only requirement is that we predict the transit time, which is easily accomplished. This oscillator has perhaps the simplest frequency equation possible. For a transit time oscillator, as we have already stated, the frequency equation is

```
fT = k                                                                    (6)
```

where f is the frequency, T is the transit time, and k is a constant for a given stage or oscillation mode. A transit time oscillator is defined for the purposes of this paper as an oscillator whose frequency is predicted by this equation. There is a sequence of values of k, with each stage or oscillation mode having its own unique value of k. k is the number of whole periods of the oscillation in the transit time T plus a fraction which is any excess of fT over an integral number of periods. If T is approximated by T = X/v(0), this equation becomes very similar to the empirical equations adopted by previous theorists. The philosophies are very different however, since equation 6 demands a realistically determined transit time taking account of jet slowing. The slowing of the jet turns out to be critical.

With the assumption that the edgetone oscillator is a transit time feedback oscillator, analogous to electronic oscillators, explaining edgetones theoretically is reduced to two independent problems, predicting the fT product sequence, or values of k, as a function of stage number, and predicting the transit time T of a jet particle from aperture to edge. In making these predictions we will assume that the longitudinal motion (x-motion) and transverse motion (y-motion) of the jet particles are independent of each other. That is, we treat the oscillation as a perturbation to the motion of the jet. This procedure turns out to be successful.

The essence of the pertubational method of solving a problem is to take a somewhat similar problem with a known solution and to make small changes in the problem statement or the solution to adapt the known solution to the new problem. The special feature of the perturbational approach is that it takes advantage of what is already known about a somewhat similar physical problem. There are many possible variations in executing the method. This perturbation procedure is too well known in physics to require extensive justification. Perturbation methods are exceedingly common in modern theoretical physics. If a calculation is too difficult to be done exactly, a perturbation calculation frequently can give a very reliable result.

The only possibly justifiable objection to be raised against this procedure for our problem is that a perturbational calculation of the motion of the jet particles is not warranted and will not be successful. But that is a speculation that can only be resolved by testing the procedure. The validity of the perturbational calculation, or more properly its usefulness, in any particular case is to be judged by its success or failure, that is, by whether or not the calculation results in an explanation in agreement with established facts and available experimental data. This is the prime criterion by which the validity of a perturbation calculation is to be judged. The inevitable assumptions involved are considered justified if the procedure is successful, otherwise not.

It is possible for our problem to dispense with predicting the fT product sequence theoretically, and to determine the sequence empirically by simply calculating T using current theories of jet slowing, and then multiplying the experimental values of f in each stage by their corresponding T values. In this paper we will develop the fT product sequence by both methods independently, empirically using Brown's experimental data and the equations of jet slowing from the published literature, and theoretically using the theory being developed in this paper. The empirical approach will be shown first. The empirical result is that indeed each oscillation mode or stage has its own unique value fT. Therefore the edgetone oscillator is indeed a transit time oscillator. This will now be demonstrated.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Transit Time T of a Jet Particle

In determining the transit time of jet particle across the aperture to edge gap, we will start with Newton's fundamental law of mechanics:

```
"mass times acceleration equals force"
```

When applied to a unit volume element of a fluid, and assuming an incompressible fluid of unit density, this becomes

$$\mathbf{r}'' = \mathbf{f} + \mathbf{g} \qquad (7)$$

where **r**, **f**, and **g** are vector quantities. **r** is the vector giving the position of the volume element, and the total force on the element has been decomposed into body forces **f** and surface forces **g**. Prandtl and Tietjens (Ref. 11) present this equation as the fundamental equation of hydrodynamics and show how it leads to the Navier-Stokes equation. The reader is referred to their book for the details of this development.

We will adopt equation 7 as our fundamental equation. The density of the fluid is immaterial in our treatment of the problem so the density is not made a parameter of our problem. By assuming that the slit, jet, and edge are infinite in both z-directions, we eliminate any consideration of the z-direction from this equation of motion of a jet particle. In keeping with our perturbation calculation, we consider that the y-

motion of the jet particles when oscillations are occurring is a minor perturbation to the motion of the jet particles which has no effect on their x-motion and in turn is not affected by the x-motion. Our adoption of this perturbation method of treating the edgetone problem allows separate treatment of the x-motion and y-motion of the jet particles. We will consider the problem of the x-motion first. But we do not have to restate and solve equation 7 as it applies to the x-motion of the particles. This problem has already been solved for us in the published literature.

(Instead of using standard mathematical notation, I will write some following expressions as they would appear if written and intended to be executed in a Basic language program. In particular I will use * to indicate multiplication, and ^ to indicate exponentiation, and the standard rules for the order of evaluation are assumed.)

The transit time of a jet particle across the gap from aperture to edge when there are no oscillations will be taken as a sufficient approximation to the transit time when oscillations are occurring. The jet will be assumed turbulent. For narrow gaps this assumption even if wrong will cause no significant error in the calculated transit time and for wide gaps the jet is almost surely turbulent, as observed by Jones. What we are doing in this perturbational approach is to assume that the motion of the jet particles when oscillations are occurring is in most respects very similar to the motion of the jet particles in a non-oscillating turbulent jet, which latter problem has already been solved. The transit time of the non-oscillating turbulent jet can be calculated from the equations of jet motion for a turbulent two-dimensional free jet given in Schlichting's text on boundary layer theory (Ref. 12, pp. 605-607). For the velocity v(x) of a jet particle at the distance x from the jet aperture, we can deduce from Schlichting's discussion of the jet motion the equation

```
v(x) = v(0)/{[1 + (x/x0)]^(1/2)}                              (8)
```

Since equation 8 is just a restatement of Schlichting's results with a different choice for the origin of coordinates, we shall regard this equation as Schlichting's and attribute it to him. We have shifted the origin of coordinates from the hypothetical source point of the jet (assumed to be the origin of coordinates in Schlichting's equations) to the jet aperture where the velocity of the jet particles is known to be v(0). Equation 8 has also been specialized to the velocity of the jet at its centerline. x0 in equation 8 (Schlichting uses a different symbol for this, the letter small s) is defined from Schlichting's equations. In our equation 8, it is the distance before the jet aperture of the hypothetical source point of the jet. From the general equations given by Schlichting, it can be shown that

```
x0 = 3 s b/4 = 5.75 b                                        (9)
```

where s in our equation 9 (Schlichting uses the Greek letter small sigma) is an empirical constant with the value 7.67 which is given by Schlichting, and b is the slit width of the jet aperture. This x0 is defined by the condition that the momentum flux for the jet determined just out of the jet aperture is b*v(0)^2. This momentum flux is assumed by Schlichting to be conserved and to be the momentum flux at any position x reached by the jet particles. The fluid density cancels out as a factor in Schlichting's equations in deriving this result for x0. The jet behavior does not depend upon the density of the fluid, or upon whether the fluid is a liquid or a gas. This is actually our justification for disregarding the fluid density in stating equation 7.

Equation 8 shows that the slowing of the jet particles is a function of the ratio X/b of gap width X to slit width b. Therefore X and b are both of equal importance in determining the degree to which the jet particles have slowed in crossing the aperture to edge gap. Equation 8 shows that only when the gap width X is small compared to the parameter x0 can the velocity of the jet particles be considered constant all across the gap from aperture to edge.

Schlichting's equation (our equation 8) for the jet particle velocity applied to Brown's data shows that for a typical entry in Brown's data the velocity of the jet particles at the edge position is less than half of the initial velocity of the jet particles at the jet aperture. Previous attempts to explain edgetones have not given

attention to this extreme slowing of the jet particles.

From equation 8 the time increment dT required for a jet particle at position x to travel the distance increment dx is

```
dT = dx/v(x) = [dx/v(0)]*[1 + (x/x0)]^(1/2)
(10)
```

Integrating this equation with the initial condition that T = 0 when x = 0, we obtain for the time T(x) required for a jet particle to reach the distance x the result

```
T(x) = T0*{[1 + (x/x0)]^(3/2) - 1}                                    (11)
```

where

```
T0 = (2/3)*[x0 / v(0)]                                                (12)
```

When x is much smaller that x0, then equation 11 reduces to T(x) = x/v(0), so that the transit time T(X) can be adequately approximated by T(X) = X/v(0), but this approximation is not generally valid. For example, for many entries in Brown's experimental data the gap width X is many times larger than the parameter x0.

The equation for T(x) can be solved for x to give

```
x(T) = x0 * {[1 + (T/T0)] ^ (2/3) - 1}                                (13)
```

where x(T) is the distance traversed by a jet particle in the time T.

Equations 8 through 13 apply only to a plane jet of thickness b at the jet aperture. The equations applying to jets with a different geometrical configuration would be different, as is evident from Schlichting's discussion of this problem of jet slowing.

Equations 8 through 13 are either taken directly from Schlichting or follow directly from Schlichting's work. They are in essence Schlichting's equations. These equations are developed by Schlichting using hydrodynamics theory and are in full agreement with and violate no fundamental tenets of modern fluid dynamics and hydrodynamics. They involve no empirical elements or approximations in fluid dynamics theory introduced by the present author. These equations are not the result of a perturbational approach by Schlichting, but Schlichting has made the usual well established approximations of boundary layer theory. The value of the empirical constant *s* in equation 9 is given by Schlichting in his book, published many years before the development of the present paper. The comparison in Schlichting's text of theoretical and experimental results validates Schlichting's equations. These equations from Schlichting will be applied to determine an empirical fT product sequence. It is here that our procedure adopts the features of a perturbation calculation. We are assuming that the occurrence of oscillations would at most make only a small difference in the time required by the jet particles to cross the aperture to edge gap, or to reach any given position x in the gap. The assumption seems reasonable. The only legitimate test of our assumption will be whether or not the predictions we make about the behaviour of the edgetone oscillator agree with CarriÃ¨re's and Brown's experimental data.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Empirically Determined fT Product Sequence, fT = k

The most extensive compilation of experimental data is that of Brown. The data of Brown's Table 1 will be exhibited later in our Tables 2 and 3. For each entry in Brown's Table 1, using Brown's b, v(0), and X as inputs to our equation 11, the transit time T to the edge was calculated. This value of T multiplied by the frequency f that Brown observed for that case gives the results tabulated in our Table 1.

Table 1. The empirical fT Products for Brown's Table 1.

| Stage Number | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| f | fT, Experimental | | | |
| 20 | -- | 1.95 | 1.17 | 0.48 |
| 100 | 3.20 | 1.97 | 1.12 | 0.45 |
| 150 | 3.11 | 2.12 | 1.22 | 0.50 |
| 1200 | 3.31 | 2.24 | 1.24 | 0.47 |
| 2400 | 3.29 | 2.21 | 1.31 | 0.51 |
| Average | 3.23 | 2.10 | 1.21 | 0.48 |

The experimental values of fT in Table 1 are nearly constant for a given stage. We conclude that the edgetone oscillator is indeed a transit time oscillator. The near constancy of the experimental values of fT for a given stage establishes empirically that the edgetone oscillator is a transit time oscillator. Our Table 1 justifies the empirical fT products

$$fT = 0.50,\ 1.25,\ 2.25,\ 3.25 \qquad (14)$$

for the oscillation stages 1, 2, 3, 4

It is quite natural to postulate an extended sequence

$$fT = 0.50,\ 1.25,\ 2.25,\ 3.25,\ 4.25,\ 5.25,\ \ldots \qquad (15)$$

for the oscillation stages 1, 2, 3, 4, 5, 6, ...

We will now show that this empirical sequence can be explained by a simple extension of our theoretical procedure.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Theoretically Predicted fT Product Sequence, fT = k

It will be assumed that the edgetone oscillator is an ordinary feedback oscillator that can be explained in a manner exactly analogous to how electronic feedback oscillators are explained. A plausible feedback mechanism and feedback loop will be defined, and a search made for the phase condition on the feedback loop that results in the positive feedback that is necessary for continuous oscillations.

We have treated the x-motion of the jet particles taking full account of the principles of hydrodynamic theory by our adoption of Schlichting's equations for the x-motion of a turbulent jet. So we are left only with the problem of applying equation 7 to determine the y-motion of the jet particle.

The only body force **f** acting on the unit volume element is gravity which is negligible for this problem and can be neglected. If frictional forces in the y-direction are neglected, which is reasonable in our approximation because these forces are very small and are certainly small in comparison to the pressure difference across the jet, the only surface force **g** we have to consider in specializing equation 7 to the y-motion is the pressure drop across the jet in the y-direction.

The plane through aperture and edge divides all space into two halfspaces, an upper and a lower. Because of the mirror symmetry about the x-z plane, we assume, using the terminology of electronic oscillators, that this is a push-pull oscillator. This is a critical assumption of the present theory. It implies that the pressure fluctuations on the upper and lower sides of the jet should be 180 degrees out of phase when oscillations are occurring. The jet particles of necessity respond to the pressure difference on the two sides of the jet. We postulate that, driven by the alternating pressure difference between the two halfspaces which acts on the jet particles, the jet particles are directed in alternating puffs into the upper and lower halfspaces. This creates the alternating pressure difference between the two halfspaces, and this alternating pressure difference controls the y-component of the jet particle motion, thereby completing the feedback loop required for continuous oscillations to exist. This is exactly analogous to how the ordinary electronic push-pull oscillator is explained, with pressure variations in the acoustic oscillator being taken as the analog of voltage variations in the electronic oscillator. A similar mechanism has been suggested before [Jones (Ref. 8)], but was not followed to the point of producing a successful theory. With the oscillator identified as an acoustic feedback oscillator, and the feedback loop defined, we look for the phase condition on the feedback loop that reinforces the excitation and makes the oscillations self-sustaining.

Assume the pressure difference across the jet varies sinusoidally with time. If the pressure acting on the lower surface of the jet is +sin(wt), then the pressure acting on the upper surface is -sin(wt), since the symmetrical geometry of the jet-edge system dictates a push-pull oscillator. The net force acting to accelerate the jet particle in the positive y-direction is then 2*sin(wt), but we will neglect the amplitude factor 2 and take this force as just sin(wt). Since the aperture to edge gap distance in the usual case is a small fraction of the wavelength of the tone produced by the edgetone oscillator, we assume that this pressure difference has the same value all across the gap at any given instant of time. (The wavelength of the tone produced, c/f, where c is the velocity of sound in the fluid, should not be confused with the wavelength seen in the jet wave.) This sinusoidal pressure difference is the dominant influence on the transverse or y-motion of the jet particles.

Define w as 2*pi*f , t as clock time, e as the time a particular fluid particle is emitted from the aperture, and T as (t - e). T is thus the length of time that a particle emitted at the time e has been traveling since it was emitted from the aperture. For the usual Greek letter pi, I have substituted the word pi, and for the usual Greek letter omega I have substituted the letter w. The quantity f is the frequency of oscillation. The differential equation for the motion of the jet particles in the y-direction, a simplification of equation 7 fully consistent with Prandtl and Tietjen's discussion of this equation, q.v., is therefore

```
y'' = sin(wt)                                                    (16)
```

with the initial conditions that y = y' = 0 when t = e or when T = 0. These initial conditions simply mean that the jet particles immediately out of the jet aperture have no displacement or velocity in the y-direction. Their velocity just out of the aperture of course in the x-direction is v(0). The primes indicate differentiation with respect to time. Any constant amplitude factor has been taken as unity in this equation since it would not affect the argument that will be made. Therefore the density of the fluid has not been considered in this equation. Any accretion or entrainment of mass by the jet as the jet particles traverse the aperture to edge

gap has been neglected. The justification of this neglect will be the agreement of theoretical predictions with experimental facts.

It is with this equation that we have introduced the effect of feedback into our theory. This equation gives the effect of the oscillations in pressure upon the motion of the jet particles thereby completing the feedback loop, since it is the motion of the particles that resulted in the oscillations in pressure.

The only features in the use of equation 16 to determine the y-motion of the jet particles that identifies our procedure as perturbational are that we are assuming that frictional forces acting on a jet particle in the y-direction are negligible and that the y-motion of the jet particles is small enough that the x-motion of the particles is not significantly altered by the occurrence of oscillations. For small enough amplitude of the oscillations this will certainly be the case. We are also assuming that the y-motion is not affected by the x-motion, which again should be true for small oscillations. So we have every reason to expect that our procedure, although perturbational in nature, should in the limit of small oscillations give an almost exact answer, and should for any oscillations give a very useful answer.

Integrating equation 16 with the given initial conditions, we obtain the solution

$$y(wt, we) = (wt - we)\cos(we) + \sin(we) - \sin(wt) \qquad (17)$$

Again any constant factor common to all terms on the right side of the equation has been ignored. From equation 17 we can derive other valid forms:

$$y(wT, we) = (wT)\cos(we) + \sin(we) - \sin(wT + we) \qquad (18)$$

$$y(wt, wT) = (wT)\cos(wt - wT) + \sin(wt - wT) - \sin(wt) \qquad (19)$$

$$y(wt, wT) = (wT)\cos(wt - wT) - 2\sin(wT/2)\cos[wt - (wT/2)] \qquad (20)$$

It should be noted the parameter T is just another way of specifying a position x in the gap (see equation 13), so that as T increases x has also increased. Our equations for the x- and y-displacement of a jet particle are both given as functions of the parameter T, that is, we are defining the path of the jet particles and the resulting jet waves parametrically.

There are two frequently used and equally valid ways of dealing with wave motions in material bodies. One way is to define and work with the wave description from the start in handling the problem, and a second is to start with the motions of the individual particles making up the wave. The description in this paper of the motion of the jet starts with the individual fluid particles of the jet. It is the individual fluid particles that move. The particle motion is the reality. In a sense the wave and the motion of the wave are illusions. In our problem each particle travels its own path essentially unaffected by the motion of the other particles, but the overall view is that we have a wave carrying out its own motion. The wave is a composite made up of the particles, each particle of the wave having been emitted from the aperture at a different time. In many cases one mode of description of what takes place may have advantages over the other mode, but both descriptions have equal validity. In this paper it was found easier to deal with a description of the motion of the individual particles rather than deal with a description of the wave from the start, but the end result is a description of the wave. The equations for y(wt, wT) are descriptions of the wave. But it must be remembered that the x-part of the wave motion is described independently by equation 13 above for x(T). For any fixed instant of time t, the parametric equations for x(T) and y(wt, wT) as a function of T, together give a snapshot of the jet wave in the aperture to edge gap at the fixed time t.

This description of the jet wave is in agreement with every precept of fluid mechanics, subject only to the qualification that it is the result of a perturbational calculation which must be validated by comparing its

predictions with experimental data. The approximations made in doing the calculation however are quite reasonable and usual approximations so there are no obvious *a priori* reasons to doubt the results of this perturbational approach. But the conclusive test will be the comparison of predictions with experimental results.

The present theory does not predict or give consideration to the amount of energy lost by the jet particles to the acoustic oscillation, although it does take account that an interaction between the jet particles and the oscillations does occur which causes the oscillations to take place. Most of the energy loss of the jet particles is due to the entrainment of mass by the jet as it crosses the aperture to edge gap, and Schlichting's equations can be used to predict this loss. The theory as presented implicitly assumes that the efficiency of conversion of jet particle energy to oscillation energy while large enough to ensure the occurrence of oscillations is small enough that no significant additional velocity loss results because of the occurrence of oscillations, in conformity with a perturbation calculation which assumes that the interaction is small. However this does not affect the validity of our calculation of the oscillation frequency, but it does mean that we can say nothing about the amplitude of the oscillations or what conditions are necessary for oscillations to begin. Since the theory does predict edgetone frequencies quite well, the *a priori* assumption that the edgetone oscillator is a low efficiency oscillator is presumably justified.

At this point in our exposition it is probably not amiss to state that every approximation known to the author in this present treatment of the edgetone oscillator has now been identified and its possible importance assessed. This in marked contrast to past treatments of this oscillator. For example, in Brown's and Powell's treatment of Brown's data and in the usual treatment of the edgetone oscillator there are two major perhaps unrecognized, but certainly unacknowledged and hidden, simplifications of the physical situation.

The jet particles are assumed in past work to hold their velocity just out of the aperture all across the aperture to edge gap. This is an unstated assumption which ignores an actual velocity loss for Brown's usual case of more than 60 percent in crossing from the aperture to the edge. There is certainly no reasonable justification for ignoring a velocity loss of this magnitude in attempting to explain this oscillator. This velocity loss of more than 60 percent by the jet particles in crossing the aperture to edge gap means that the jet particles in these cases have also lost more than 60 percent of their initial kinetic energy before they reach the edge in crossing the gap. The velocity loss is the result of the jet gaining mass by accretion as the jet particles cross the aperture to edge gap with the momentum flux of the jet at any position in the gap the same as its value at the jet aperture. The conservation of the momentum flux is a key feature of Schlichting's equations. It follows immediately that the fraction of the initial kinetic energy lost by the jet is the same as the fraction of the initial velocity lost. This large energy loss strongly suggests that it is not hypothesized interactions of the jet wave at or with the aperture or edge that are the important factors in producing edgetones but that it is instead the very real interaction of the jet particles with the oscillating acoustic pressure field sin(wt) as they cross the gap from aperture to edge that gives rise to the oscillations. This latter hypothesis is adopted in this paper as the explanation of edgetone production. The important interactions occur not at either the aperture or edge but in the space between aperture and edge, since that is where most of the energy loss of the jet particles occurs. The position of the edge is very important, of course, since a boundary condition imposed on the jet wave at the edge will be the condition that determines the frequency of the jet-edge oscillations.

Additionally by calculating the number of wavelengths in the jet wave in the gap as "gap width divided by wavelength measured at the edge" it was tacitly assumed in past work that the wave length is constant across the gap and independent of position in the gap. There is neither theoretical or experimental justification for this assumption. The possible effects of this assumption in trying to explain edgetones were not assessed. Perhaps the existence of this assumption was indeed not recognized. But it causes a serious error in the phenomenology assumed for the oscillator. Theorists believed there were more wavelengths in the jet wave in the gap for each oscillation mode than actually existed. This mistaken belief led both Curle and Powell to postulate equation 5 above for the edgetone oscillator frequencies.

The two gross simplifications of the true physical situation just identified led to another false conclusion that the phase velocity of the jet wave was a small fraction of the jet particle velocity. These errors are probably the major reason for past failures to explain edgetones.

Equation 18 gives us the path in the aperture to edge gap of a particle identified by its time of emission e from the aperture. Equation 19 will give us the values of k in equation 6 which define the expected edgetone frequencies, and equation 20 will give us information about the phase velocity of the jet wave in the aperture to edge gap. Equations 19 and 20, coupled with Schlichting's equations from standard fluid dynamics theory for the slowing of a turbulent jet, will allow us to predict with remarkable accuracy the frequencies and phase velocities that Brown observed in his experiments. These predictions are made without introducing any empirical or *ad hoc* factors into the theory to force a fit to the experimental data.

The kinetic energy lost by the jet is the energy source for the edgetone oscillations. From a simple assumption concerning this energy loss a frequency condition can be derived which turns out to be in agreement with the experimental data. Assume that the instant of most rapid increase of pressure in a halfspace coincides with the instant of the greatest rate of loss of kinetic energy by the jet to that halfspace. The assumption will be justified by the success of the resulting theory. For a sinusoidal pressure variation, the instant of most rapid increase of pressure occurs as the pressure changes from negative to positive values and goes through zero. The instant of the greatest rate of loss of kinetic energy by the jet to a halfspace should be the instant of greatest mass commitment to the halfspace.

We see immediately that if the distance X from the jet aperture to the edge is such that the resulting transit time T of a jet particle from the aperture to the edge is such that fT = 0.500, a maximum net mass commitment to a halfspace results. The jet particles at the aperture always switch from one halfspace to the other in phase with the driving pressure and continue to flow into the halfspace for just a halfperiod, so a maximum net commitment of mass to the halfspace results if the distance to the edge is such that jet particles that are emitted from the aperture just as the pressure sin(wt) across the jet equals zero reach the edge distance one halfperiod after they are emitted from the aperture. This gives us a first value fT = 0.500 which we predict will result in oscillations.

But a greatest net commitment of mass to a given halfspace should occur also as the jet particles switch simultaneously at both the aperture and the edge out of that halfspace, instead of switching just at the aperture, as is the case for fT = 0.500. Jet particles just out of the aperture always switch from one halfspace to the other in phase with the driving pressure, that is, y(wt, wT) for the value wT equal to zero changes from negative to positive values as the driving pressure sin(wt) changes from negative to positive values, so we require that the same be true for the value of wT at the edge distance if oscillations are to occur. The driving pressure sin(wt) is zero and switches from negative to positive values when wt equals zero or a multiple of 2*pi, so y(wt, wT) at the edge must also be zero and switch from negative to positive values when wt equals zero or a multiple of 2*pi. This boundary condition which we have just imposed reduces equation 19 to an expression which we predict is a condition on wT at the edge that will result in oscillations.

```
wT = tan(wT)                                          (21)
```

This equation is easily solved with a programmable electronic calculator or computer, but it is also a well known equation whose solutions are tabulated in many texts or collections of mathematical data. The solutions meeting our criteria are the alternate zeroes of this equation. The solutions of interest lead to

```
fT =  1.230, 2.239, 3.242, 4.244, 5.245, ...          (22)
```

taking into account that

```
wT = 2*pi*fT.                          (23)
```

We have now made the prediction that the observed values of fT which will result in edgetone oscillations should occur in the sequence

```
fT = 0.500, 1.230, 2.239, 3.242, 4.244, 5.245, ...          (24)
```

for the stage numbers 1, 2, 3, 4, 5, 6, ... .

The boundary condition that determines the position of the edge when oscillations are occurring is that the mass committed to a half-space be a maximum when the acoustic driving pressure in that halfspace has its fastest rate of increase.

The theoretically predicted fT sequence of equation 24 should be compared with the experimentally (or empirically) established sequence of equation 15. The agreement is almost exact.

The sequence 22 gives the frequencies of best operation of the ordinary electronic transit time oscillator known as the klystron. This can be deduced from the analysis of the klystron oscillator given by Marcuse (Ref. 13). It is an elementary exercise to deduce from Marcuse's equation (4.3-14) on page 137 of his book that equation 21 above predicts the frequencies of best performance of the oscillator Marcuse designates as a Transit Time LC Oscillator. The procedure followed here in deriving equation 24 is quite similar in principle to the procedure used by Marcuse in deriving his equations for the klystron oscillator and indeed the resulting frequency equations can be shown to be identical except for the additional term fT = 0.500 in equation 24. Marcuse's derivation is certainly more sophisticated than the derivation here, and additionally his oscillator is not a push-pull oscillator such as the edgetone oscillator obviously is, and allowance has to be made for this difference. Although the details of our derivations differ on the surface, the fundamental idea of the two derivations is the same. In the klystron oscillator the electrons in interacting with the oscillating electric field either lose energy to the electric field, in which case oscillations continue, or gain energy from the field in which case oscillations die away. In the edgetone oscillator the jet particles in interacting with the oscillating acoustic field sin(wt) in the aperture to edge gap either lose energy to the acoustic field, in which case oscillations occur, or gain energy from the field in which case oscillations die away. Which of these two things happens depends upon the phase relation between the motion of the particles and the oscillating field. In the electronic oscillator depending upon the phase of the electric field oscillation when a particular electron enters the region where the electric field acts on it, the electron is either speeded up or slowed down; the result is a bunching of the electrons in the electron stream, with the final effect produced on the oscillator output depending upon the transit time of the electrons through the active field region. In the edgetone oscillator depending upon the phase of the acoustic field pressure oscillation when a particular jet particle enters the region where the acoustic field acts on it, the particle is either directed into the upper half-plane or the lower half-plane; the result is the production of a wave in the jet particle stream, with the final effect produced on the oscillator output depending upon the transit time of the jet particles through the active field region. In both oscillators the transit time of the particles must be such that the appropriate phase relation to result in the build up of the oscillating field, electric or acoustic, exists. This sets a condition that determines the oscillation frequency. I provisionally assumed the validity of my equation 19, and I then assumed a phase relation between the jet wave and the acoustic field which I thought was very probably necessary if oscillations were to occur, which gave me equation 21 which leads in straightforward fashion to equation 24, and I justify my assumptions by the end result, which is the complete agreement of predictions with Brown's data. Agreement with the experimental data is ultimately the only way any physical theory is justified and is the usual way of establishing the validity of a perturbational approach to a problem.

[Note added in March, 2001: Although the present theory recognizes the work done by the moving jet particles against the acoustic field to be the cause of the oscillations, it does not attempt to calculate the amplitude of the acoustic field oscillations that will result. This is not necessary to predict the edgetone

oscillator frequencies. However, such a calculation quite analogous to the calculation made by Marcuse for the klystron oscillator in deriving his equation (4.3-14) would appear, at least in principle, to be a straightforward addition to the present paper.]

The theoretical fT sequence of equation 24 for practical purposes differs only trivially from the empirical fT sequence of equation 15. Therefore our theory has predicted the experimentally established fT product sequence of equation 15. Equation 24 immediately accounts for the numerical factor of 2 required in the empirical equations 1 through 4 to fit the experimental data for stage one. This is evident upon approximating T by X/v(0) for stage one. This is not an adequate approximation for wide gaps, and consequently not usually adequate for the higher stages of oscillation, and perhaps not always adequate for stage one. By "wide gap" in the context of this paper is meant that the gap width X is very much greater than the slit width b. The slowing of the jet in crossing the gap is very important for wide gaps, amounting to more than one-half in the usual case as will be shown later. Equation 24 automatically makes provision for the effects of jet slowing.

Equation 24 gives the basic prediction of the present theory. This prediction is purely theoretical and has no empirical or *ad hoc* elements introduced to force a fit to the experimental data. Since this equation is in agreement with the experimental data, as shown by Table 1, it is established theoretically as well as empirically that the edgetone oscillator is a transit time oscillator. The transit time T is the controlling parameter determining edgetone frequency. The system parameters b, v(0), and X are of only incidental importance insofar as they determine T. All sets of these parameters giving the same value of T will produce the same frequencies f. Since the parameter T determines the frequency of the edgetone oscillator, the slowing of jet particles must be explicitly taken into account in any theory of the oscillator. The failure to do this is obviously the major failing of past theoretical efforts.

The transit time T as shown by equation 11 is a function of b, v(0), and X only; it is independent of the stage number of the oscillation occurring. Therefore equation 24 predicts that the ratio of the two frequencies seen in two different stages for the same values of b, v(0), and X in both stages is a function of the two stage numbers only and is independent of the particular values of b, v(0), and X. The combination of equations 11 and 24 of this paper predicts this fact and offers the explanation for it.

Brown's empirical equation for the edgetone frequency (our equation 4 above)was determined by a process that conforms to the condition on b, v(0), and X of the preceding paragraph. Brown's values for the empirical factor a in his equation for stages 2, 3, and 4 were found by successively multiplying the value a = 1.0 for stage 1 by the frequency ratios found for stage 2 to stage 1, stage 3 to stage 2, and stage 4 to stage 3, where each of these ratios was found for the same values of b, v(0), and X in the two stages for which the ratio was being determined. The values of v(0) and X were not the same in the determination of each ratio but were the same in determining the two frequencies involved in a ratio. The value of b was the same in all of Brown's frequency measurements. Brown's process is valid although it does compound the errors in the individual frequency ratios as they are multiplied together to determine the successive values for a. However, we can recover Brown's original experimentally determined frequency ratios free from any compounding of errors by just taking the ratios of the successive values of a in his empirical equation. For the successive frequency ratios for stage 2 to stage 1, 3 to 2, and 4 to 3, Brown's equation gives the respective values 2.3, 1.65, and 1.42. Curle's and Powell's equation (our equation 5) predicts the successive ratios 1.80, 1.44, and 1.36, which are rather different from Brown's values. Our equation 24 predicts the values 2.46, 1.82, and 1.45, which are close to Brown's values. It should be noted that the fact that Brown's equation predicts these frequency ratios correctly does not mean that the equation predicts frequencies correctly, that is, that the equation gives the correct dependence of frequency upon system parameters. The equation ignores the effect of slit width b upon edgetone frequency. All that would be required to make Brown's equation grossly in error in predicting frequencies while still predicting frequencies ratios correctly would be to use a greatly different value of b in Brown's experimental set-up while keeping v(0) and X the same. The frequencies observed experimentally would change since the transit time T(X) changes when b changes, so Brown's equation would predict the wrong frequencies, but it would still predict the frequency

ratios correctly (within experimental error, since equation 4 is an empirical equation based upon experimental data).

It will be noted from equation 11 that for sufficiently large values of slit width b (that is, when X is small compared to x0) the transit time T is effectively independent of b, and is a function of v(0) and X only. For this condition, the frequency of oscillation can be considered to be independent of the value of b and to be a function of v(0) and X only. For this condition the transit time T is very closely approximated by T = X/v (0). However, this is not generally true; when X is large compared to x0, then the value of b becomes important, and equation 11 must be used to determine the transit time.

There are two conditions to be satisfied if oscillations are to be produced in a feedback oscillator. One is a condition on gain or amplitude around the feedback loop, and the other is a condition on the phase change around the feedback loop. Since neglecting all amplitude factors in setting up and solving equation 16 makes it impossible to discuss the required gain in the oscillator, the amplitude factors in equations 17 through 20 have no absolute significance. But since we are doing a perturbation calculation, we have tacitly assumed that the maximum amplitude (i.e., y-displacement) reached by the wave while crossing the aperture to edge gap is some indefinite small value, large enough however to sustain the oscillations. The relative magnitudes of the y-displacement as wT increases, that is, as the wave traverses the aperture to edge gap, are very important though as the equations show that the amplitude of the wave increases as it traverses the gap, which is a well recognized feature of the jet-edge oscillator, a feature now predicted by our theory.

The phase condition ordinarily fixes the frequency of the oscillation. The phase condition is what we addressed when we assumed that the instant of fastest increase in the pressure of the acoustic field in a given halfspace coincided with the instant of greatest rate of loss of kinetic energy by the jet particles to that halfspace. This condition applied to our equation 19 gives equation 21 as a condition to be satisfied by the oscillator, which leads to equation 22, a condition on the fT product if oscillations are to occur. It is easily seen however that the condition fT = 0.500 not predicted by equation 21 also satisfies our assumption that the instant of the fastest increase of pressure in a halfspace coincides with the instant of the greatest rate of loss of energy by the jet particles to the halfspace. Adding this term to our fT sequence of equation 22 gives the final prediction of the fT sequence presented in equation 24.

For all but stage one the edge position is at a zero crossing of the jet wave y(wt, wT), defined by equations 19 and 20, for wt equal to zero or a multiple of 2*pi, but the basic assumption does not demand that all edge positions coincide with a zero crossing of this jet wave, this just happened to result in all stages but the first. There is no reason from the theory offered to conclude that this is a necessary condition for stage one. So there is no valid reason to doubt the prediction for the first stage just because one of the related circumstances is somewhat different. Every term in this final predicted fT sequence is a consequence of a single intuitively obvious assumption: the instant of fastest increase of pressure in a halfspace coincides with the instant of greatest rate of loss of kinetic energy by the jet particles to that halfspace.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## An Extended Comparison of Theoretical Predictions and Experimental Data

It is immediately apparent without numerical calculation, by approximating equation 11 for large and small values of b and x, that the predictions of edgetone behavior from equations 11 and 24 are in qualitative agreement with every aspect of the experimental behavior of edgetones pointed out above in Figure 1. For fixed b and X, the frequency is predicted to vary directly as v(0). For fixed b and v(0), the frequency is predicted to vary inversely as X for small X, and inversely as the three-halves power of X for large values of X. The fact that the jet is turbulent automatically introduces the three-halves power of X necessary to fit the

experimental data for wide gaps. For fixed v(0) and X, the frequency is predicted to increase as a function of b for small values of b, and to approach a limit as b becomes large, in agreement with CarriÃ¨re's data. The parameter b although universally neglected in previous theoretical attempts to explain edgetones is seen to be as important as the parameter X in determining the edgetone frequency. The constancy of the ratio of frequencies observed in different stages for the same set of parameters b, v(0), and X, is predicted. The empirical equations 1 through 4 for stage one are predicted by the first term of the sequence 24. In every case the predicted behavior is in qualitative agreement with the observed experimental facts shown in Figure 1. Additionally it will be shown later that the theory predicts the phase velocities of the jet wave near the edge that Brown measured.

It will now be shown that the theory is in almost exact quantitative agreement with the available experimental data. The best collections of experimental data are those of Carriere and Brown. The critical test of any theory is to predict the results of their experiments. The most extensive compilation of experimental data is that of Brown. The data of Brown's Table 1 will be exhibited in our Tables 2 and 3.

It has already been established by our Table 1 that the experimental values fT in each oscillation stage are in close agreement with the theoretical values in equation 24. For a further comparison with the experimental data of Brown's Table 1, the gap width X for each entry in Brown's table was calculated as a function of stage number, b, v(0), and f. The transit time T was first found from the sequence 24 using the stage number and observed frequency f. The gap width X was then predicted using equation 13. The comparison of the theoretical and experimental values of X is shown in our Table 2.

Table 2. Comparison of the theory with Brown's experimental data. The slit width b is 0.1 cm in all cases. With the stage number, b, v(0), and f as inputs to the theory, all gap widths X have been predicted. The numbers appended to X in the column headings indicate stage numbers. This table has the same arrangement of parameters as Brown's Table 1.

| | f (Hz) | X4 (cm) | X3 (cm) | X2 (cm) | X1 (cm) | v(0) (cm/s) |
|---|---|---|---|---|---|---|
| Experiment | 20 | -- | 5.68 | 3.92 | 2.02 | 137 |
| Theory | | 8.13 | 6.26 | 4.07 | 2.08 | |
| Experiment | 100 | 3.48 | 2.41 | 1.57 | 0.75 | 212 |
| Theory | | 3.51 | 2.67 | 1.69 | 0.82 | |
| Experiment | 150 | 3.33 | 2.50 | 1.64 | 0.79 | 309 |
| Theory | | 3.44 | 2.61 | 1.65 | 0.80 | |
| Experiment | 1200 | 1.74 | 1.28 | 0.79 | 0.34 | 984 |
| Theory | | 1.71 | 1.28 | 0.78 | 0.36 | |
| Experiment | 2400 | 1.58 | 1.15 | 0.75 | 0.33 | 1750 |
| Theory | | 1.56 | 1.16 | 0.71 | 0.32 | |

The agreement of the theoretically calculated values of the gap width X with Brown's experimental values is excellent. The calculation of Brown's Table 1 shown in our Table 2 does not critically depend upon the theory used to predict the theoretical fT sequence of equation 24. Our Table 1 justifies an empirical fT sequence fT = 0.50, 1.25, 2.25, 3.25, ..., which is so close to the theoretical sequence of equation 24 as to make little difference in calculating Brown's Table 1 using equation 13, with T from this empirical sequence

rather than from equation 24. It is therefore established empirically, as well as theoretically, that the edgetone oscillator is a transit time oscillator. To challenge this conclusion is to challenge not just the theory behind the predicted fT product sequence of equation 24, but to challenge either Brown's experimental data or Schlichting's equations for the slowing of a turbulent jet, since these two things alone suffice to give the fT product sequence and to predict Brown's Table 1. The validity of equation 24 is now established as an empirical fact, which is independent of any theory leading to that equation.

Every conclusion about the edgetone oscillator that will be stated in this paper is derivable just from Brown's data and Schlichting's equations, and is therefore an empirical fact. Objecting to the theory of this paper will not dispose of these conclusions since the conclusions can be established empirically independently of the theory of this paper. The principal equations 11, 13, and 24 of this paper, with the ancillary equations 9, 12, and 23, suffice to describe the edgetone oscillator, and do not depend upon anything in this paper for their validity.

The theory can be compared with Brown's data in other ways. Table 3 shows the comparison of the predicted values of the frequency with the experimental values. The transit time T was calculated for each of Brown's cases using equation 11, and the frequency was then found substituting this T into equation 24. The predicted frequencies are within a few percent of the experimental values.

Table 3. Comparison of the theory with Brown's experimental data. The slit width b is 0.1 cm in all cases. The first line in each group of three lists the experimental values of gap width X and initial jet velocity v(0) that give the experimentally observed frequency in the second line. The corresponding theoretically calculated frequencies are given in the third line.

| Stage Number | 4 | 3 | 2 | 1 | v(0) (cm/s) |
|---|---|---|---|---|---|
| Gap Width X, cm | -- | 5.68 | 3.92 | 2.02 | 137 |
| f experimental, Hz | -- | 20 | 20 | 20 | |
| f theoretical, Hz | -- | 23 | 21 | 21 | |
| Gap Width X, cm | 3.48 | 2.41 | 1.57 | 0.75 | 212 |
| f experimental, Hz | 100 | 100 | 100 | 100 | |
| f theoretical, Hz | 101 | 114 | 110 | 111 | |
| Gap Width X, cm | 3.33 | 2.50 | 1.64 | 0.79 | 309 |
| f experimental, Hz | 150 | 150 | 150 | 150 | |
| f theoretical, Hz | 157 | 159 | 151 | 152 | |
| Gap Width X, cm | 1.74 | 1.28 | 0.79 | 0.34 | 984 |
| f experimental, Hz | 1200 | 1200 | 1200 | 1200 | |
| f theoretical, Hz | 1180 | 1200 | 1210 | 1270 | |
| Gap Width X, cm | 1.58 | 1.15 | 0.75 | 0.33 | 1750 |
| f experimental, Hz | 2400 | 2400 | 2400 | 2400 | |

| | | | | |
|---|---|---|---|---|
| f theoretical, Hz | 2370 | 2440 | 2250 | 2340 |

........................................................... ........

Our equations do fit and predict Brown's experimental data. The agreement of the theoretical predictions with Brownâ€™s experimental data is excellent, within a few percent. The theory has no empirical or *ad hoc* elements introduced to force a fit to Brownâ€™s experimental data. Brown's data cover a very wide range of parameters, a factor of 120 for frequency f, 12.8 for initial jet velocity v(0), and 17.2 for gap distance X. The agreement of the theoretical predictions with Brown's experimental data is too close to allow a conclusion that the agreement is a fortuitous accident. It will be shown later that Brown's data on the phase velocity of the jet wave are also predicted.

Brown's data for the lower frequencies and wider gaps give values of the ratio (X/x0) appreciably greater than one, so therefore our equations 11 and 24 predict that the edgetone frequency in these cases should be exhibiting an inverse three-halves power dependence upon gap width X. That our equations predict Brown's data shows that this is the case. Brown's data therefore confirm what CarriÃ¨re and Jones both observed, that for some conditions the frequency varies inversely as the three-halves power of the gap width X.

The close agreement that has just been demonstrated of the predictions from this perturbational treatment of the edgetone problem with Brown's detailed experimental data proves the validity of our perturbational treatment. This was actually to be expected since the assumptions made in adopting this treatment are quite reasonable simplifications of the usual full hydrodynamic approach.

The theory will next be compared with CarriÃ¨re's data giving the frequency f as a function of slit width b with other parameters fixed. Unfortunately CarriÃ¨re gave the blowing pressure rather than the initial jet velocity so we are faced with the problem of converting blowing pressure to initial jet velocity. The conversion is not straightforward. In the general case where only the blowing pressure is known, Bernoulli's equation can not be relied upon to give an accurate prediction of the jet particle velocity. For Bernoulliâ€™s equation to apply it is necessary that the jet flow be steady, frictionless, and along a streamline, and that fluid density be a function of pressure only (Ref. 14). These conditions are not always, or even usually met. For a Borda tube aperture it has been shown theoretically that the initial jet velocity is 50 percent of the value predicted by Bernoulli's equation, for a sharp edged orifice the jet velocity is typically about 63 percent of the Bernoulli value, and for a Venturi orifice it can be close to 100 percent (Refs. 15). From the apparatus diagrams given by CarriÃ¨re, conversion factors of about 50 to 63 percent seem appropriate, and conversion factors in this approximate range give good agreement with our theoretical predictions while higher values would not. This conversion factor is an empirical factor we are forced to introduce in order to compare the theory with CarriÃ¨reâ€™s data.

Our Table 4 shows the comparison of theoretical predictions with CarriÃ¨re's data, using 66.4 percent as the conversion factor in determining the jet velocity from CarriÃ¨re's pressure values. The comparison was made in two ways. First, the theoretical values of T(X) were calculated using equation 11 and the resulting fT products were found for comparison with the predicted values of fT. The agreement with the value 0.500 predicted for stage 1 is almost exact, establishing stage 1 as the oscillation mode for Carriere's data. All of CarriÃ¨re's data appear to be for oscillations in stage 1. Second, using the calculated values of T(X) and assuming stage 1 oscillations, the expected frequencies were predicted using equation 24. The theory gives the right variation of frequency with slit width b for a 20 to 1 variation in b. Any error in the velocity conversion factor would appear as a constant scaling factor for the experimental fT products and the theoretically calculated frequencies.

Table 4. Comparison of theory with CarriÃ¨re's data giving edgetone frequency f as a function of slit width b. The gap X was fixed at 13.40 cm. The blowing pressure was 10 cm of water. The jet velocity is taken as 2690 cm/sec, or 66.4 percent of the value from

Bernoulli's equation.

| b (cm) Experiment | fT Exp. | fT Theory | f (Hz) Exp. | f (Hz) Theory | Error (percent) |
|---|---|---|---|---|---|
| 1.00 | 0.509 | 0.500 | 70.4 | 69.1 | -1.8 |
| 0.90 | 0.506 | 0.500 | 68.0 | 67.2 | -1.2 |
| 0.80 | 0.495 | 0.500 | 64.4 | 65.1 | 1.0 |
| 0.70 | 0.503 | 0.500 | 63.0 | 62.6 | -0.6 |
| 0.60 | 0.501 | 0.500 | 59.8 | 59.7 | -0.2 |
| 0.50 | 0.490 | 0.500 | 55.2 | 56.3 | 2.0 |
| 0.40 | 0.464 | 0.500 | 48.4 | 52.1 | 7.7 |
| 0.30 | 0.476 | 0.500 | 44.6 | 46.8 | 5.0 |
| 0.20 | 0.457 | 0.500 | 36.4 | 39.9 | 9.5 |
| 0.10 | 0.501 | 0.500 | 29.6 | 29.5 | -0.2 |
| 0.05 | 0.597 | 0.500 | 25.6 | 21.4 | -16.3 |

The velocity conversion factor was an empirical factor we had to introduce to apply the theory to CarriÃ¨re's data. However the theory then predicts CarriÃ¨re's results with very good accuracy as shown by the column giving the percentage error in the predictions.

CarriÃ¨re's experimental data show that the slit width b is fully as important as the parameter X in determining the edgetone frequency and that the slit width b should be an important parameter in any theory of the edgetone oscillator. CarriÃ¨re's data show that with v(0), X, and stage number fixed, a variation of slit width b by a factor of 20 to 1 caused a variation in edgetone frequency by a factor of 2.75 to 1. All previous theorists have failed to consider the dependence of edgetone frequency upon slit width b.

It is important to consider CarriÃ¨reâ€™s experiments in which the frequency varied inversely as the three-halves power of the aperture to edge distance. This is what the present theory predicts for wide gaps and a turbulent jet. Jones noticed that this variation occurred when the gap was wide and the jet was turbulent. Table 5 shows the comparison with CarriÃ¨re's data. T(X) was calculated using equation 11 for each value of X given by CarriÃ¨re. Then, assuming stage 1 oscillations, the expected frequencies were predicted using equation 24.

Table 5. Comparison of the theory with CarriÃ¨re's data for which the frequency varied inversely as the three-halves power of the gap width. The oscillations are identified as stage 1. The blowing pressure was 16 cm of water. The jet velocity is taken as 2455 cm/sec, or 48 percent of the value from Bernoulli's equation. The slit width is 0.25 cm. All numbers are CarriÃ¨re's except the theoretical values for f in the third column and the percentage values of the prediction errors.

| Experiment X (cm) | Experiment f (Hz) | Theory f (Hz) | Error (percent) | Experiment f*[X^(3/2)] |
|---|---|---|---|---|
| 16.7 | 28.2 | 29.2 | 3.5 | 1920 |
| 15.9 | 30.0 | 31.3 | 4.3 | 1900 |

| | | | | |
|---|---|---|---|---|
| 14.9 | 33.3 | 34.3 | 3.0 | 1910 |
| 13.9 | 35.2 | 37.8 | 7.4 | 1820 |
| 12.9 | 42.6 | 42.0 | -1.4 | 1970 |
| 11.9 | 47.6 | 47.0 | -1.2 | 1960 |
| 10.9 | 54.0 | 53.1 | -1.7 | 1940 |
| 9.9 | 62.0 | 60.6 | -2.3 | 1960 |
| 8.9 | 74.0 | 70.1 | -5.3 | 1940 |
| 8.1 | 80.0 | 79.6 | -0.5 | 1860 |

.................................................................

The agreement shown is excellent as shown by the small percentage errors in the error column. The empirical factors used in converting blowing pressures to blowing velocities are within the range commonly found necessary (see Refs. 15) in applying Bernoulli's equation for this purpose.

The success of the present theory in predicting the results of critical edgetone experiments confirms this theory, and offers confirming evidence for the theory of jet motion presented in Schlichting's book. It suggests a new technique to determine the velocities of jet particles as a function of slit width b, initial jet velocity v(0), and distance X from the jet aperture.

These data of CarriÃ¨re's demonstrate conclusively that for some conditions the edgetone frequency varies inversely as the three-halves power of the gap width X. Brown's data and Jones' finding also confirm this. All previous theoretical efforts have ignored this strong three-halves power variation of frequency with gap width.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Comparison of Theoretical Predictions with More Recently Available Experimental Data

The author recently (summer, 2000) found on the Internet some very interesting experimental data on edgetones made available by Professor Yoshikazu Suematsu of Nagoya University, apparently abstracted from a published paper (Ref. 21), not yet seen by the present author. His data are very important since they greatly expand the range of v, f, X, and possibly aperture slit width b, covered by the easily available published data on edgetones. Although not stated in the limited material on the Internet, it appears that the fluid in his experiments was a liquid, but the theory of this paper applies to liquids as well as to gases. His curves published on the Internet give data for a single velocity v = 2.5 cm/sec showing frequencies varying from about 0.125 Hz to slightly over 0.375 Hz in stages 1, 2, and 3, for values of aperture to edge distances varying from about 3 cm to 20 cm. Unfortunately, the value of the slit width b corresponding to this data is not given. It will be assumed here that the data are for a single value of the slit width. Under the assumption that the theory of this paper is correct, a value for b, or equivalently for the parameter x0 of our theory, that fits all of Suematsu's data points, is easily deduced from a single data point. This fit to Suematsu's data justifies these assumptions.

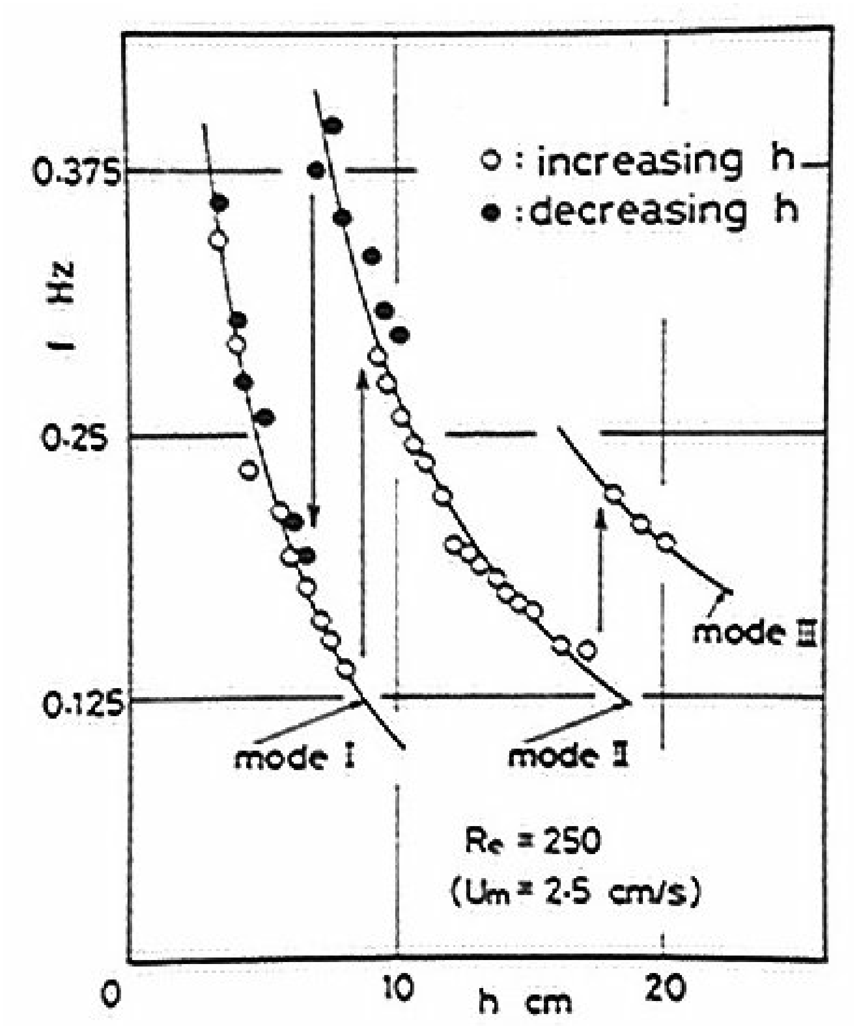
This material can be found at the Internet addresses:

http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/
http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/ET.html
http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/ET_fr.jpg

Our Figure 2 shown here is Suematsu's graph that presents his data that was just referred to. This graph as

shown here is found at the third of the addresses just given, and is found at the second address greatly reduced in size.

# Figure 2




Note that Suematsu uses the conventional symbol "h" for the aperture to edge distance where the present author uses the symbol "X". Table 6 shows the data for selected points as read by the author from Suematsu's curves in Figure 2. All entries in the table are Suematsu's experimental data except the entries labeled "f, our theory" and "f, Powell". Applying the theory of this paper to the case Stage II, v equal 2.5 cm/sec, X equal 18 cm, and f equal 0.125 Hz, we calculate the value 9.23 cm for the parameter x0 of our theory, or equivalently the value 1.61 cm for the slit width b. Using this value for the parameter x0, the theory of this paper predicts the frequencies designated "f, our theory" for the other cases shown in the table, which can be compared with Suematsu's experimental values. The frequencies designated "f, Powell" were calculated using Powell's empirical equation for the edgetone frequency (our equation 5).

...........................................................

Table 6. The comparison of Suematsu's experimental data with theoretical predictions. All entries are experimental values read from Professor Suematsu's published graph, except for the calculated frequencies designated as "f, our theory" or "f, Powell". Those designated "f, our theory" were calculated by the

theory of this paper, assuming that the parameter x0 is 9.23 cm, or equivalently that the slit width b in Suematsu's experiments was 1.61 cm. Those designated "f, Powell" were calculated using Powell's empirical equation (our equation 5). [Note: the frequency entry in parentheses marked by an asterisk is the data point used to calculate x0; therefore the calculated value of frequency "f, our theory" for this case should be and is identical to the value "f, experiment".]

| STAGE | 1 | 2 | 3 | |
|---|---|---|---|---|
| v(0) | 2.5 | 2.5 | 2.5 | cm/sec |
| | | | | |
| X | 9.0 | 18.0 | --- | cm |
| f, experiment | 0.125 | (0.125)* | --- | Hz |
| f, our theory | 0.114 | 0.125 | --- | Hz |
| f, Powell | 0.174 | 0.156 | --- | Hz |
| | | | | |
| X | 5.0 | 11.0 | 16.0 | cm |
| f, experiment | 0.250 | 0.250 | 0.250 | Hz |
| f, our theory | 0.222 | 0.223 | 0.257 | Hz |
| f, Powell | 0.313 | 0.256 | 0.254 | Hz |
| | | | | |
| X | 3.0 | 8.0 | --- | cm |
| f, experiment | 0.375 | 0.375 | --- | Hz |
| f, our theory | 0.387 | 0.322 | --- | Hz |
| f, Powell | 0.521 | 0.352 | --- | Hz |
| | | | | |
| X | 10.0 | 10.0 | --- | cm |
| f, experiment | 0.106 | 0.266 | --- | Hz |
| f, our theory | 0.101 | 0.249 | --- | Hz |
| f, Powell | 0.156 | 0.281 | --- | Hz |
| | | | | |
| X | --- | 20.0 | 20.0 | cm |
| f, experiment | --- | 0.106 | 0.194 | Hz |
| f, our theory | --- | 0.108 | 0.196 | Hz |
| f, Powell | --- | 0.141 | 0.203 | Hz |

Powell's empirical equation fails very badly in all of its predictions of Suematsu's frequencies for stage 1, and in its prediction of Suematsu's frequency for stage 2 for the gap distance X equal to 20 cm.

It is evident from Table 6 that the theory of this paper predicts with very good accuracy the frequencies seen by Suematsu in his experiments. This comparison of the theory with the experimental data required that the parameter x0 of the theory be calculated using the data of one of Suematsu's data points in order to predict the frequencies seen in all the other data points. The prediction of these other frequencies is very good. However, even without a value for the parameter x0 (or of the slit width b), it is possible to compare certain predictions of the theory with Suematsu's data; this will be demonstrated.

We will recall that for the same set of parameters v(0), b, and X applied to any two stages of oscillation, our theory predicts that the ratio of the two frequencies seen in the two different stages is a function only of the

two stage numbers and is independent of the particular values of v(0), b, and X. All of Suematsu's data is for a single value of jet velocity v(0) and a single value of aperture slit width b. Therefore our theory, as presented here without any adjustments, should predict the ratio of the two frequencies that Suematsu saw in two different stages for the same value of aperture to edge gap distance X. For the aperture to edge gap distance X equal to 10.0 cm in both stage 2 and stage 1, the ratio of the frequency seen in stage 2 to the frequency seen in stage 1, as calculated from the experimental data in Table 6, is 2.51; for the gap distance X equal to 20 cm in both stage 3 and stage 2, the ratio of the frequency in stage 3 to the frequency in stage 2 is 1.83. The corresponding ratios predicted by the theory of this paper are 1.230/0.500 = 2.46 and 2.239/1.230 = 1.82. The theoretical value 2.46 is to be compared to the experimental value 2.51, and the theoretical value 1.82 is to be compared to the experimental value 1.83. The agreement of the predictions with Suematsu's data is essentially perfect. In contrast, the ratios predicted from Curle's and Powell's frequency equation for the edgetone oscillator (our equation 5) are 1.80 and 1.44, very different indeed from the ratios seen in Suematsu's experiments. As pointed out in detail in this paper, the equation of Curle and Powell is based upon a serious misinterpretation of Brown's experimental data by Brown, Curle, and Powell, and is therefore unsupported by experimental data.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Jet Wave Phase Velocity

The present theory predicts that the phase velocity of the jet wave is not a fixed quantity across the gap as tacitly assumed by Brown and accepted by later theorists, but is a function of position in the gap. Equation 20 demonstrates this most conveniently. For values of wT small enough that sin(wT) can be approximated by wT, equation 20 becomes

```
y(wt, wT) = wT [cos(wt - wT) - cos(wt - wT/2)]                (25)
```

which in turn with the same approximation for sin(wT) reduces to

```
y(wt, wT) = [(wT)^2] * sin(wt - 3wT/4)                        (26)
```

The last equation indicates that the phase velocity at any point very near the aperture is four-thirds times the jet particle velocity at that point. This is perhaps more obvious if T in the sine term of equation 26 is replaced by its equivalent as a function of X given by equation 11.

For values of wT greater than about 2, or for values of fT greater than about 0.318, the first term of equation 20 dominates its behavior and the second term can be neglected. Equation 11 is then approximated by

```
y(wt, wT) = (wT)cos(wt - wT)                                  (27)
```

This first term taken alone indicates a phase velocity at a point equal to the jet particle velocity at that point. Therefore for values of fT greater than about 0.318, the phase velocity at a point is almost equal to the jet particle velocity at that point. The phase velocity at the position x(T) in the gap should then be closely approximated by v(x) where v(x) is given by equation 8. Both jet and phase velocities are functions of position in the gap. The phase velocity of the jet wave is never less than the velocity of the jet particles. Brown's and later theorists' conclusion that the phase velocity of the jet wave is only a fraction of the jet particle velocity is wrong.

Since the jet particle velocity v(x) becomes smaller as the jet particles approach the edge, the phase velocity of the jet wave also becomes smaller as the edge is approached. This means that the wavelength seen in the

jet wave also becomes smaller as the edge is approached. Therefore Brown's conclusion that the number of wavelengths of the jet wave in the aperture to edge gap was equal to the gap width divided by the wavelength measured just before the edge is incorrect and leads to a serious overestimate of the number of wavelengths in the oscillation of the jet in the gap. This then means that Curle's and Powell's equation (our equation 5) for the frequency of the edgetone oscillator has no support in Brown's data, is therefore an unsupported conjecture, and is wrong.

These conclusions of the present theory about the phase velocity of the jet wave are very different from those of previous theorists, who neglected the slowing of the jet particles and tacitly took the jet velocity anywhere in the gap as always equal to the jet velocity at the aperture. The phase velocity of the jet wave was believed to be about half that value anywhere in the gap, as exemplified by the common interpretation of equations 3 and 4, Brown's interpretation of his experimental data, and Curle's and Powell's interpretation of equation 5.

It will not be demonstrated here but the present theory can predict without approximation the wavelengths measured by Brown, and therefore the phase velocities which Brown found. Equation 21 predicts the sequence of jet travel times T that correspond to all the zero crossings of the jet wave in the gap for the values of wt equal to zero or to some multiple of 2*pi. For the purpose of calculating wavelengths, the value T equal to zero should be taken as the first member of this sequence of values of T. This sequence of values of T substituted into equation 13 predicts the sequence of values of position x in the gap that correspond to successive zero crossings of the jet wave, with the value x equal to zero being the first value of x in the sequence. Presumably, as seen by Brown, the distance between successive zero crossings is one-half wavelength, and the distance between alternate zero crossings is one complete wavelength. Starting from the value x equal to zero, every second value of x in this sequence x of zero crossing values is a value of X in the sequence of edge positions X that give rise to edgetone oscillations in stages two and higher. The value of X for stage one is not a member of this sequence. We will arbitrarily, for convenience, include the value X equal to zero in this sequence of values X giving rise to edgetone oscillations, although of course no oscillations occur for this value of gap width X. We will designate the sequence of values of X that results by

```
X(I) = X(0), X(2), X(3), X(4), ...                                (28)
```

where, except for X(0), the value X = X(I) is a value of aperture to edge distance that gives rise to edgetone oscillations in stage I. X(1) is not a member of the sequence X(I) defined in this manner.

Define Lambda(I) to be the wavelength seen by Brown for stage I. Then for stage numbers I equal two and higher the wavelengths Lambda(I) seen by Brown are given by

```
Lambda(2) = X(2) - X(0)
(29)
```

```
Lambda(I) = X(I) - X(I - 1);  I = 3, 4, 5, ...                  (30)
```

These wavelengths Lambda(I) are just the successive differences between the values of X in the sequence X (I) of equation 28. These are the only wavelengths that Brown could define. He could not experimentally define a wavelength in stage one. The wavelengths Brown saw are effectively just the increments in aperture to edge distance X between successive oscillation stages, although Brown did not recognize this. This procedure does not give a wavelength prediction for stage one, but stage one oscillations are a special case

arising from the value fT = 0.500 not predicted by equation 20, and a zero crossing of the jet wave does not occur for the value of x (or X) which corresponds to the value of T for stage one. The derivation of the product fT for stage one indicates that we should not expect to see a complete wavelength in stage one, and indeed the experimental fact is that Brown could not define a wavelength in stage one. Therefore our theoretical procedure predicts this aspect also of Brown's data.

It is much simpler to do the equivalent of the process just outlined by taking the phase velocity of the jet wave at a point as equal for practical purposes to the jet particle velocity at that point, so that wavelengths at a point are closely approximated by dividing the jet particle velocity at that point by the edgetone frequency, except for points very near the jet aperture.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Comparison of Predicted and Experimental Values of Phase Velocity

Brown's procedure gives the phase velocity not at a point but averaged over one wavelength just before the edge. Although Brown did not state it this way, obviously what Brown determined was the ratio of the phase velocity of the jet wave averaged over one wavelength *immediately adjacent to the edge* to the jet particle velocity *at the aperture*, and stated this way Brown's experimentally determined ratios are valid numbers, that is, they are correct for Brown's particular experiment, but there is no *a priori* reason to assume that these same ratio values for phase velocity at the edge to particle velocity at the aperture would be valid for experimental parameters different from Brown's. A more obvious comparison would have been to compare the phase velocity at the edge to the jet particle velocity also at the edge, but this is not the comparison which Brown made. Brown implicitly assumed that although the two velocities were measured at different points in his experimental setup, the ratio so determined had universal validity. There is no obvious justification for such an assumption, and in fact the assumption is seriously wrong. For the higher stages, for which the wavelength is much shorter since there are more wavelengths in the gap and the jet has slowed appreciably, Brown's average is taken over a short distance just adjacent to the edge. The present theory predicts that, except very near the jet aperture, the phase velocity of the jet wave at a point is very nearly equal to the jet particle velocity at that point. Therefore, Brown's experimentally determined ratios, for the higher stages, of the phase velocity near the edge to the jet particle velocity at the aperture, should differ only slightly if at all from the theoretically calculated ratio v(X)/v(0) at the edge. And this is indeed the case. For every entry in Brown's Tables 2 and 3 our equation 8 gives v(X)/v(0) = 0.38 at the edge. The jet particles in crossing from the jet aperture to the edge have lost over sixty percent of their initial velocity. Of the nineteen valid entries in Brown's Table 2 for the ratio of phase velocity immediately before the edge to jet particle velocity at the aperture, eighteen are in the range 0.36 to 0.43, in close agreement with our prediction. One entry gives 0.47 for the ratio. (There are twenty entries in the table but one has to be discarded because it is for stage 1 and is not experimental data but is a guess by Brown, this entry should not have been included in Brown's table.) For the highest stage in Brown's Table 3, the ratio values are 0.40, 0.41, and 0.38, again in agreement with our prediction of 0.38 for what Brown would see for the higher stages. Schlichting's equation for the slowing of a turbulent jet (our equation 8) gives an accurate prediction of the phase velocities Brown observed. If Brown or others had calculated the slowing of the jet particles, they would have recognized these phase velocities as being for practical purposes just the velocities to which the jet particles had slowed. Brown's experimental data are entirely consistent with the conclusions of the present paper about the phase velocity, in fact, the present theory predicts the phase velocities which Brown found. We are forced to the conclusion that while Brown's experimental data are excellent, serious errors have been made in the interpretation of these data. Brown tacitly assumed that the jet particle velocity at any point in the aperture to edge gap was equal to the jet particle velocity just out of the aperture, and that the phase velocity of the jet wave at any point in the gap was a small constant fraction of that constant jet particle velocity. These errors of interpretation contributed to previous failures to develop a satisfactory theory.

It will be noted that the conclusion of this paper that at points not very near the aperture the phase velocity of the jet wave at those points is essentially equal to the jet velocity at those points is now an empirically established fact, not dependent upon the theory of this paper. If Brown's basic data are correct and Schlichting's equations for the slowing of the jet are correct, then this conclusion is correct. The conclusion follows from Brown's data and Schlichting's equations, neither of which depends upon anything in this paper.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Critical Review of Past Theoretical Efforts

No past theoretical effort to explain edgetones could even attempt to predict all the basic phenomenology of edgetones outlined above in Figure 1, and certainly not the detailed experimental data of CarriÃ¨re and Brown, so the only detailed comparison with past work that is possible is to discuss their inferences about the phase velocity of the jet wave in the gap. It will be shown that Brown's data on the phase velocity of the jet wave have been misinterpreted by Brown and others, and that Brown's data are predicted by the present theory.

Karamcheti, Bauer, Shields, Stegen, and Woolley in 1969 (Ref. 16) discussed briefly the availability of experimental evidence on the phase velocity of the jet wave in the aperture to edge gap. They state *"the only experimental information on the phase criterion is that indicated by Brown's measurements of the wavelength from smoke pictures of the oscillating jet."* Brown injected smoke into the jet and photographed the jet disturbance in the gap. There was not enough detail in the photographs to allow the number of wavelengths in the jet disturbance in the gap to be counted. He was able to define and measure a wavelength in the jet disturbance in these photographs only near the edge and only for the higher stages. He states that photographs of stage one were such that no wavelength could be defined. For the other stages he could only make measurements near the edge since insufficient details of structure were discernible in earlier parts of the jet path. Great accuracy should not be expected in these measurements. The measured wavelength multiplied by the frequency gave a phase velocity which was about one-fourth to one-half of the velocity Brown attributed to the jet, which was the velocity of the jet at the aperture. Brown did not tabulate the phase velocities themselves but instead gave the ratios of the phase velocities he determined to the initial jet velocity at the aperture. The ratios Brown determined are valid numbers, but Brown misinterpreted their significance. Brown's values for the wavelength are valid only immediately before the edge. There are the implicit assumptions in Brown's treatment of his data that the jet does not slow down and that the wavelength in the jet disturbance in the gap is the same at all positions in the gap, and therefore that both the jet particle velocity and the jet wave phase velocity are constant independent of position in the aperture to edge gap. Brown concluded then that the phase velocity (assumed constant) was a small fraction of the jet particle velocity (also assumed constant). There is no basis in theory or experiment for Brown's assumptions. It is known that these jets do slow down rapidly. However later theorists neglected the known slowing of the jet and uncritically accepted Brown's interpretation of his data. Karamcheti and Bauer (Ref. 17; also see Ref. 16, page 295) have noted that assuming a single disturbance wavelength and propagation velocity throughout the edgetone jet is not correct, but their remark seems to have passed unnoticed by most theorists.

There are other major problems with Brown's presentation and interpretation of his experimental data. Brown could not experimentally define a wavelength for stage one oscillations. However he assumed that for stage one the wavelength was just the aperture to edge distance, and he calculated values for phase velocity in stage one on the basis of this assumption. In his text he stated that he had done this. Unfortunately, and inexplicably, he included these assumed and calculated values for stage one in his table

of real experimentally determined values for the other stages, without an explicit warning in the table that for stage one the values given had not been experimentally determined but were in effect guesses. This entry should not have been included in Brown's table of experimental data. This is the entry in Brown's Table 2 that was rejected from consideration in the paragraph above. Brown had no experimental value of wavelength or phase velocity for stage one. Strangely, the existence of this invalid entry in Brown's table of data has not been previously pointed out. It is an unfortunate fact that past theorists, subsequent to Brown, using these data overlooked Brown's statement of what he had done and treated these particular values for stage one as valid experimental data. Curle and Powell both made this error, and each of the many subsequent investigators of edgetones uncritically accepting Brown's presentation and interpretation of his data, or following Powell's lead in this matter has unwittingly contributed to the perpetuation of this persistent error that the phase velocity of the jet wave is a small fraction of the jet particle velocity. This inattention to what Brown wrote and the resulting confusion about Brown's data have undoubtedly contributed to the failures to develop an adequate theory.

The lack of detail except near the edge in Brown's photographs made it impossible to count the number of wavelengths in the oscillation of the jet in the gap. The number assigned by Brown for stage one and entered into his table of what was otherwise numbers derived from experimental data, one complete wavelength, was a pure guess as we saw in the paragraph immediately preceding. This entry must be disregarded. Except for stage one, the numbers assigned were obtained by dividing the gap width by the wavelengths measured just before the edge, which was the only place with sufficient detail to allow the wavelengths to be defined and the measurements made. This procedure tacitly assumed that the wavelength was constant and independent of position in the gap. The measured wavelength values of course are valid but must be properly interpreted. There is neither experimental nor theoretical justification for Brown's assumption that the wavelength measured just before the edge was the wavelength at all positions in the gap. This gave much too large a result for the number of wavelengths in the gap because the wavelength in the jet gets smaller as the edge is approached since the jet is slowing down. Every entry Brown gives for the number of wavelengths in the gap must be rejected. Theorists making uncritical use of Brown's data who accepted his interpretation of that data were led to incorrect conclusions. For every stage Brown assigned one complete wavelength too many for the integer part of the number of wavelengths in the jet disturbance in the gap and the fractional part of this assignment above an integer has no validity either. Curle and Powell accepted this assignment as the basis for postulating the empirical equation they proposed (our equation 5). It is evident that the values Brown tabulates for the number of wavelengths in the oscillation of the jet in the gap have no validity whatever, and offer no confirmation or support to the equation that Curle and Powell proposed for the edgetone frequency. That equation is completely devoid of experimental support by Brown's data and is therefore an unsupported conjecture. All subsequent theoretical papers (and there are many) following Powell's lead and adopting this equation are based on the false premise that this equation is supported by Brown's experimental data. Therefore these papers must be rejected or at least critically reexamined. Brown's and later theorists' interpretation and treatment of Brown's excellent experimental data were unchallenged until the present paper.

Powell has probably been the most influential writer on edgetones since Brown, and certainly the most prolific. He has been the major champion since Brown of the viewpoint that the phase velocity of the jet wave is everywhere a small fraction of the jet particle velocity. Powell has continued to publish in various journals numerous articles and letters on edgetones (too many to reference all here, but see for example Ref. 18) offering elaborations of the viewpoint and equation he first presented in 1953. Although Powell's equation (equation 5 above) is obviously inadequate and cannot predict or explain the detailed data of Brown, and certainly not the data of CarriÃ¨re, Powell has continued to staunchly support this equation and his original approach to this problem. The influence of Powell upon other theorists has been very great, which in most respects is unfortunate. The continued acceptance of Brown's assumptions and interpretation of his data by Powell vitiates most of Powell's work on edgetones. Powell's approach to the problem of edgetones and his equation have been adopted (as recently as 1998, see Ref. 20) as a starting point by many later theorists attempting to explain edgetones. It will be apparent that the present paper completely invalidates Powell's and his followers' approach to the problem of edgetones. (Note added in 1999: Powell's

and his followers' efforts to explain edgetones, continuing to depend upon Brown's assumptions and interpretation of his data, now cover a period of almost fifty years, and have yet to produce either a theory or an empirical procedure that can predict the experimental data of CarriÃ¨re and Brown.)

It truly seems that the major reason for the long failure to have a successful theory of edgetones has been the uncritical acceptance by substantially all theorists after Brown of the faulty interpretation Brown offered of his data. Powell and following theorists just did not consider that Brown might have been seriously wrong in his treatment of his data. However even a cursory reading of Brown's paper shows that the treatment and interpretation that Brown offers of his experimental data are dependent upon two unstated assumptions, that the jet particles do not slow down in crossing the aperture to edge gap, and that the wavelength of the jet wave measured immediately before the edge is the wavelength at all positions in the aperture to edge gap. The existence of these unstated and incorrect assumptions should have been recognized long ago. Brown certainly gives enough information about how he interpreted his experimental data to allow these unwarranted assumptions to be easily identified. But since Brown's paper, nearly every paper on edgetones or the organ flue pipe has tacitly assumed as a starting point that the jet particle velocity does not vary across the gap, and that the phase velocity of the jet wave in the gap is only a fraction of the jet particle velocity in the gap. These papers are immediately seriously in error.

To summarize, the phase velocity of the jet wave in the edgetone oscillator and in musical instruments such as the flute and organ flue pipe is not a small fraction of the jet particle velocity, as stated to be the fact in most texts and papers dealing with this oscillator and these musical instruments. To slightly adapt here a short but very appropriate quote from T. Needham written about a different subject but for a similar situation (T. Needham, Visual Complex Analysis, Oxford University Press, 1997, p. 386): *"We have made a fallacy of an assertion that is to be found in most texts. Perhaps the mere frequency with which this myth has been reiterated goes some way to explaining how it has acquired the status of fact."*

Theorists attempting to explain edgetones have also neglected large parts of the known phenomenology of edgetones. They have not given attention to the finding of CarriÃ¨re, confirmed by Jones and also deducible from Brown's data, that for some conditions the edgetone frequency varies inversely as the three-halves power of the gap width X, or to the finding of CarriÃ¨re, confirmed by Brown, that the edgetone frequency depends upon the aperture slit width b. These dependencies are too strong to be ignored in an adequate theory of the edgetone oscillator. No prior theoretical paper known to the author gives serious consideration to CarriÃ¨re's work, acknowledges that a three-halves power dependence of frequency on gap width X exists which must be explained, or makes the slit width b an important parameter of the theory proposed. None takes account of the possible effects of jet slowing upon edgetone production.

In many of the prior theoretical papers on edgetones vortices shed from the jet wave at the aperture, traveling across the aperture to edge gap, and interacting with the edge have been assumed to be the causative agent giving rise to the edgetones. None of these papers however has presented a quantitative explanation of just how this postulated interaction gives rise to edgetones or presented a theory that can predict the experimental data on edgetones, so it is not established as a fact that vortices are the causative agent of edgetone production or even a significant factor in edgetone production. That has remained a theoretical speculation. No mention of vortices as contributing to edgetone production has been found necessary in the present paper. The causative agent for edgetone production has been identified in this paper as the interaction between the moving jet particles and the oscillating acoustic pressure field of the oscillations. That is a very satisfactory outcome as it reduces the explanation of edgetones to the same explanation that accounts for the operation of comparable electronic oscillators. Ultimately the explanation of the operation of any electronic oscillator is the interaction of an electric current with an oscillating electric field. An analogous explanation seems to hold true for any acoustic oscillator. The current has to interact with the field in order to add energy to the field, which is a necessary condition if oscillations are to be produced. The production of vortices may indeed be a phenomenon that accompanies the production of edgetones, but the vortices do not appear to be the cause of the edgetones, they are instead an accompaniment or consequence of edgetone production. In support of this we note that Schlichting's

equations for a turbulent jet which are certainly valid when no oscillations are occurring give no indication that vortices are present or being produced. All that is necessary to convert Schlichting's equations for the turbulent jet to our equations for the jet wave of the edgetone oscillator is to introduce the oscillating acoustic pressure field *sin(wt)*.

The theory of the present paper has aroused considerable opposition among those imbued with the prevailing opinion expressed in virtually all past work that the phase velocity of the jet wave is only a fraction of the jet particle velocity. It is evident that the prevailing opinion is based on the two unstated assumptions that (1) the jet particle velocity is constant everywhere in the gap and always equal to its value at the aperture, and (2) the phase velocity of the jet wave is constant everywhere in the gap and always equal to its value measured just adjacent to the edge. There is no basis for the tacit assumptions that these velocities are constant across the gap, and therefore that the ratio of phase velocity to jet particle velocity is a constant independent of position in the gap which is equal to the ratio of phase velocity just adjacent to the edge to the jet particle velocity just out of the aperture. In the eighty years since Krueger's paper no one adopting his view of the phase velocity of the jet wave has been able to predict the observed data on edgetones. The present theory with no *ad hoc* elements does predict Brown's and CarriÃ¨re's data on edgetones with exceedingly good accuracy, including Brown's data on the phase velocity of the jet wave.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Recommendation for Modern Experiments

It is a strange fact that present day theorists discussing edgetones still must base their discussions mainly upon the experimental results of Brown published (as of the year 1999) more than sixty years ago, in what we might consider to be "the dark ages" of instrumentation development and certainly still "the dark ages" of our understanding of edgetones. Also there is an important lack in Brown's data. Although it was known from CarriÃ¨re's prior work on edgetones that the aperture slit width b was just as important as the initial jet velocity v(0), gap width X, and the stage number in determining the edgetone frequency f, Brown did not vary the slit width b in his reported data. The result has been that modern theorists hardly give attention to the experimental parameter b as being important in determining the edgetone frequency. I know of no prior paper on edgetones other than CarriÃ¨re's that recognizes the great and equal importance of the aperture slit width in determining the edgetone frequency.

Modern experiments on edgetones, guided by our present understanding of edgetone phenomenology such as is exhibited in this paper, and using modern technologies would be very useful. A repetition of Brown's experiments, with the slit width b being one of the parameters varied in addition to v(0), X, and the stage number should be carried out. The range of variation of the parameters could also be increased with no great difficulty using modern technology. Although actually Brown's data have been shown in this paper to be sufficient to verify the present theory, new experiments designed specifically to test the theory would put the matter beyond cavil.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Summary

This paper establishes as an empirical fact independent of any theory offered in this paper that the edgetone oscillator is an acoustic transit-time oscillator governed by the equation fT = k where T is the transit time of

a jet particle from the jet aperture to the edge, and k is a constant that depends upon the oscillation mode. The values of k are established by presently available experimental data. The paper also offers a theory that predicts the value of k seen for each oscillation mode.

The theory presented here predicts with extremely good accuracy the available experimental data on edgetones without the introduction of any empirical factors intended to force a fit to that data. It has the virtue that it shows that the edgetone oscillator is just another simple oscillator of well known type, explainable by the same principles of oscillator operation that govern a multitude of familiar electronic oscillators. The only critical feature in the presentation is the adoption of a perturbational approach to the problem, and that approach has been fully validated by the result.

This theory was developed in the years 1971 through 1973. A brief account of the work was presented at the Los Angeles meeting of the Acoustical Society of America, 30 October-2 November, 1973; and an abstract (Ref. 19) of that presentation was published in 1974. The key equations of this paper, equations 9, 11, 12, 13, and 24, are presented in the abstract. The abstract's statement that the phase velocity of the jet wave in stage one is twice the jet particle velocity there is incorrect. The error arose from the hasty assumption that only the second term of equation 20 was important for very small values of wT. (The phase velocity of the jet wave at a point very near the aperture is four-thirds times the velocity of the jet particles at that point.)

A first version of this paper was written in early 1972. The present version was written in 1974. Revisions were made in 1999 and 2000. The original paper stated only casually that the oscillation would be considered as a perturbation to the motion of the jet. A major purpose of the recent revisions was to emphasize that the theory being presented was a perturbation theory, and to show that this approach was fully justified. Text to that purpose has been added. No equations and no conclusions were changed. One new equation, equation 7, was introduced. The literature since early 1974 has not been reviewed. A detailed account of the theory has not been previously published.

The most important conclusion of the present paper is that the edgetone oscillator is an acoustic transit time oscillator, not different in principle from many electronic oscillators. Perhaps equally or even more important is that erroneous assumptions in Brown's presentation and interpretation of his excellent experimental data are identified. These assumptions tacitly adopted by Brown and unquestioningly accepted by later theorists relying upon Brown's paper have had the most serious consequences upon efforts to develop an understanding of the edgetone oscillator.

(Note added in 1999: One recent theorist on edgetones, Young-Pil Kwon (Ref. 20), in two interesting papers has independently noted in referring to Brown's paper that *"the jet velocity decreases with distance along the jet axis"* and that *"the wavelength measured near the edge tip may be shorter than the average wavelength along the stand-off distance"* in consonance with this paper. Although not greatly emphasized by him, this amounts to a denial of Brown's assumptions about the jet and phase velocities. He did adopt Powell's equation (our equation 5) as a starting point for his discussion of the edgetone problem, but recognized that as a guide to edgetone oscillator behavior this equation would require major modifications.)

Although usually unstated, implicit in most previous theoretical efforts to explain edgetones is the tacit assumption that the jet velocity remains constant all across the gap from aperture to edge. It is usually also explicitly assumed that the jet wave is characterized by a constant phase velocity which is a small fraction of that assumed constant jet velocity. The first assumption is contradicted by the known facts about jet behavior, and the second assumption is without support in experiment or theory. No theory based on these assumptions has had success in predicting the data from basic edgetone experiments, despite a plethora of theoretical efforts extending over most of a century. None gives even a qualitative prediction of all the basic phenomenology of edgetones outlined above.

None of these previous theoretical efforts has given attention to or attempted to explain either qualitatively or quantitatively the three-halves power variation of frequency with gap width seen in some experiments, or

the strong variation of frequency with slit width which is known to exist and for which quantitative data is available. The slit width is not even a parameter of these theories. Despite claims of validity that have been made for these theories, none has been successfully applied to predict the data of CarriÃ¨re and Brown. These data exist, their validity has not been challenged, and a successful theory must explain them. The author regards the predictions of these data as the basic tests of any edgetone theory. The theory of the present paper is the only theory of edgetones yet developed that predicts and explains the results of these basic experiments.

The theoretical situation is further clouded by the fact that a clear distinction is not always made by theorists between the theory of edgetone oscillations and the theory of flute or organ pipe oscillations. An adequate theory of the organ flue pipe will have little application to the edgetone oscillator. The impact of the Q-factor of the flute's or organ pipe's resonant air column upon the oscillation frequency of the musical instrument is so strong that a clear distinction between the two theories should be made. Any theory of edgetones has direct bearing upon the theory of flutes or organ pipes since the edgetone appears to be the exciting agent of the flute or organ flue pipe. However the Q-factor of the musical instrument so modifies the behavior of the edgetone oscillator as to make separate discussions of the two oscillators at least desirable if not strictly necessary. (See the discussion of Lord Rayleigh's data on the variation of frequency of an organ flue pipe with changes of blowing pressure, which can be found at http://www.nmol.com/users/wblocker/index.htm where the present paper is also found. Lord Rayleigh's data shows that changes of blowing pressure, and consequently of blowing velocity, that would more than triple the frequency of an edgetone oscillator make a change of only a few percent in the frequency of an organ flue pipe oscillator. This stabilization of the frequency is attributable to the Q-factor of the organ flue pipe. Even modest values of the Q-factor have very great effects in stabilizing the frequency sounded by the pipe. Any parameter having an influence this great upon the frequency sounded by the flue pipe should receive great attention in any theory of the flue pipe. Most theories of the organ flue pipe oscillator fail to mention the pipe's Q-factor and the few that do give scant attention to its importance.)

The assumption that the edgetone oscillator is a transit time oscillator leads immediately to a theory that predicts all the basic phenomenology of edgetones and gives numerical predictions in almost exact agreement with all critical experimental data. This is the first theory that can predict in detail the experimental results of Brown and CarriÃ¨re. But the theory is not essential. Every important conclusion of this paper follows from Brown's experimental data and Schlichting's equations for the slowing of a jet. These alone suffice to establish empirically the fT product sequence, thus to establish empirically that the edgetone oscillator is a transit time oscillator, and to establish empirically that, except possibly very near the aperture, the phase velocity of the jet wave at a point is equal for practical purposes to the jet particle velocity at that point. These things are now empirically established facts, and are independent of the theory that led to them.

The major factor leading to this theory was the conviction that the edgetone oscillator must be an ordinary feedback oscillator which could be explained in the same fashion that all other feedback oscillators are explained. With this conviction, the feedback loop is easily identified and analyzed. In fact Jones had already identified the feedback loop and the feedback mechanism. The edgetone oscillator is an acoustic feedback oscillator, more specifically a transit time oscillator. The calculation of the transit time must take account of the slowing of the jet particles and take account that the jet is turbulent. The same principles that explain the operation of ordinary electronic feedback oscillators suffice to explain the operation of this oscillator.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## References

(1) C. Sondhaus, Ann. Phys. 91, 128 & 214 (1854)
(2) J.M.A. Lenihan and E. G. Richardson, Phil. Mag. 29, 400 (1940)
(3) W. Koenig, Phys. Z. 13, 1053 (1912)
(4) E. Schmidtke, Ann. Phys. 60, 715 (1919)
(5) F. Krueger, Ann. Phys. 62, 673 (1920)
(6) M.Z. CarriÃ¨re, J. de Phys. et Rad. 6, 52-64, (1925)
(7) G.B. Brown, Proc. Phys. Soc. 49, 493 & 508 (1937)
(8) A. T. Jones, J. Acoust. Soc. Am. 14, 127(A) (1942/1943)
(9) N. Curle, Proc. Roy. Soc. A216, 412 (1953)
(10) A. Powell, Acoustica 3, 233 (1953)
(11) Prandtl and Tietjens, Fundamentals of Hydro- and Aeromechanics, Chapter XV, pp. 251 et seq., Dover Publications, Inc., New York, 1957
(12) H. Schlichting, Boundary Layer Theory, McGraw-Hill Book Co., New York, 4th edition, 1960, third printing (with limited revisions), p. 596 & pp. 605-607
(13) D. Marcuse, Engineering Quantum Electrodynamics; Harcourt, Brace and World, Inc.; New York, 1970, pp. 127-140. (A second augmented edition of this book, published in 1980 by Academic Press under the title Principles of Quantum Electronics, by Marcuse, also contains this material.)
(14) V. L. Streeter, Fluid Mechanics, McGraw-Hill Book Co., New York, 1960, p. 88
(15) J. R. D. Francis, A Textbook of Fluid Mechanics, Edward Arnold (Publishers) Ltd., London, 1958, Chapter 8; J. M. Kay, An Introduction to Fluid Mechanics and Heat Transfer, Cambridge University Press, Cambridge, 1963, Chapter 2; P. S. Barna, Fluid Mechanics for Engineers, third edition, Plenum Press New York, 1969, Chapter 6
(16) K. Karamcheti, A. B. Bauer, W. L. Shields, G. R. Stegen, and J. P. Woolley, National Aeronautics and Space Administration publication NASA SP-207, pp. 275-304 (esp. pp. 288, 295), (1969)
(17) K. Karamcheti and A. B. Bauer, SUDAER Rept. 162, Dept. of Aeronautics and Astronautics, Stanford Univ., (1963)
(18) A. Powell, J. Acoust. Soc. Am. 33, 395 (1961); 34, 163 (1962); many others
(19) W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)
(20) Young-Pil Kwon, J. Acoust. Soc. Am. 100 (5), November 1996, pp. 3028-3032; J. Acoust. Soc. Am. 104 (4), October 1998, pp. 2084-2089
(21) Y. Suematsu, Oscillating Phenomena of a Jet in a Jet-edge system; Bull of JSME, 251-15, p. 1479

Return to the Table of Contents

Return to The Home Page of Wade Blocker.



Comments will be appreciated.

wblocker@nmol.com

# A Theoretical Model

# for the

# Edgetone Oscillator


## Wade Blocker

wblocker@nmol.com


March, 2004

This paper presents and rationalizes a simple model for the edgetone oscillator, the development of which led to the paper "The theory of the edgetone oscillator." That paper is available as the lead paper on the web site where this paper is found. It is recommended that that paper be read in order to understand every point discussed here. This paper is being presented to clarify and further rationalize the model on which I based "The theory of the edgetone oscillator."

The edgetone oscillator ceased to be of interest to mainstream physicists over sixty years ago which is probably the reason it has remained unexplained. It appears that everyone now working in the field starts with the misconception that the phase velocity of the jet wave in the oscillator is a small fraction of the jet particle velocity because that is what everybody keeps on saying, parroting each other, a misconception that these workers seem very reluctant to give up. But a review of the origins of this erroneous belief shows that it is not justified by the available data, and shows in fact that it is seriously wrong. Part of the problem is that they don't define very well just what they mean when they state their belief. This problem is discussed in my paper.

The author of "The theory of the edgetone oscillator" has brought it to the attention of a few authors of books or papers that discuss the edgetone oscillator. It is the opinion of this author that none of these present a correct view of the relation between the phase velocity of the jet wave and the velocity of the jet particles. They don't even seem to recognize that the jet particles slow down markedly as they cross the aperture to edge gap in this oscillator. This makes it impossible for them to develop a successful theory of the edgetone oscillator.

None of these has openly challenged the validity of the application in my paper of Schlichting's equation for the slowing of the jet particles to Brown's experimental data, from which every conclusion of mine about the edgetone oscillator can be shown to follow without the interposition of any further theoretical considerations. Nevertheless a simple theory can be presented that still requires Schlichting's equation to complete the application of the theory but predicts a relation between the phase velocity of the jet wave and the velocity of the jet particles that is made independently of Schlichting's equation, but which prediction the application of Schlichting's equation to Brown's data verifies. It also predicts that the edgetone oscillator is a transit time oscillator, which prediction the application of Schlichting's equation to Brown's data also verifies.

The authors don't like the theory I present for the jet wave in the gap and challenge my conclusions on that basis, ignoring the fact that my conclusions don't depend upon the theory of the jet wave that I present. The conclusions can be and were drawn from my theory of the jet wave, but they are independent of that theory. As I point out in the paper under discussion they can be drawn just as well on a purely empirical basis from Schlichting's equation applied to the available experimental data, particularly to that of Brown. To challenge my conclusions requires that either Schlichting's equation or Brown's data, or both, be shown to be incorrect. This has not been done and presumably cannot be done.

The critical assumption of my theory of the jet particle motion in the gap is that the oscillator is a push-pull oscillator. Everything follows from that assumption. I assume that the edgetone oscillator is a push-pull oscillator because there is mirror symmetry about the x-z plane through the aperture and the edge. In the common electronic oscillators mirror symmetry in the circuitry results in a push-pull oscillator, so I assumed that mirror symmetry results in the same thing for the edgetone oscillator. That means the driving forces on the jet particles are effectively 180 degrees out of phase on the two sides of the stream of particles. My equations for the jet wave follow in straightforward fashion from that assumption. I don't have to justify this assumption *a priori*, the assumption has instead the *a posteriore* justification that it leads to predicted results that are in agreement with the experimental facts. Actually this how all theories in physics have been established, they have been postulated first and then justified by the agreement of their predictions with the experimental facts. It is not even necessary that the initial assumptions be correct. It is just necessary that they incorporate enough of the truth that they lead to valid conclusions. Some theorists discussing the edgetone or the organ flue pipe, professional physicists even, seem to forget this fundamental fact at times.

You don't have to have a mental picture of how things work in physics, although

of course people are much more comfortable about things when they do have such a picture. The most successful physical theory in existence is quantum mechanics, and there is no one in the world who has a mental picture of how it works, no one can claim to understand it, although there are innumerable people who know how to apply it. The results that quantum mechanics predicts are in agreement with the experimental facts and therefore quantum mechanics is accepted as a correct theory. No exceptions to its predictions have yet been found. The equations I present for the edgetone oscillator suffice to predict all the basic experimental results of operating an edgetone oscillator and by the same standards that apply to quantum mechanics you would have to say that I have a correct theory. There are no competing theories that can predict the experimental results. This is the first theory developed that can predict Brown's and Carriere's experimental results for the edgetone oscillator.

The edgetone oscillator is still in the area of classical physics so a mental picture of its operation is surely possible, but it is not necessary. However for the edgetone oscillator a qualitative picure or model of its operation is easily developed and a theory based on this qualitative picture will give quantitative predictions that are in agreement with all the available experimental data.

If the 180 degree phase difference postulated above and in my paper "The theory of the edgetone oscillator" actually exists then the particles in the jet stream in alternate half cycles of the edgetone oscillation should be directed in alternating puffs into the upper and lower halfspaces, that is directed alternately above and below the plane of mirror symmetry. Such an alteration in particle motion should cause a corresponding shift in where the center of the pressure wave is. The center of pressure should be alternately above and below the plane of symmetry. That shift in position justifies the idealization, or simplification if you will, that there are pressure variations on the the two sides of the stream of jet particles (that is, above and below the jet stream in my presentation) that are 180 degrees out of phase. The assumed pressure difference gives rise to a predicted particle motion and this predicted particle motion gives rise to the assumed pressure difference on the two sides of the jet stream. That means that the feedback loop exists which every oscillator with a continuous output requires. This picture does not have to be demonstrated *a priori* to be correct although one hopes that it is and indeed it appears that it might be, its real justification is *a posteriore*, that it leads to predictions in agreement with the experimental facts. Such agreement is the justification for the use of the assumptions.

The predictions deduced from the model are in complete agreement with the experimental facts so the assumptions of the model are justified.

Return to The Home Page of Wade Blocker.

Comments will be appreciated.

wblocker@nmol.com

# The Development and History

# of

# "The Theory of the Edgetone Oscillator"


## Wade Blocker

wblocker@nmol.com



September, 2003


It is recommended that the paper "The theory of the edgetone oscillator," available at the same web site as this present history of the paper, be read before this paper. That will make clear all the points discussed in the present paper.

This theory was developed in the years 1971 through 1973. A brief account of the work was presented at the Los Angeles meeting of the Acoustical Society of America, 30 October-2 November, 1973; and an abstract of that presentation was published in 1974 [W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)]. The key equations of the paper under discussion, equations 9, 11, 12, 13, and 24, are presented in the abstract. The abstract's statement that the phase velocity of the jet wave in stage one is twice the jet particle velocity there is incorrect. The error arose from the hasty assumption that only the second term of equation 20 of the paper was important for very small values of wT. (The theory predicts that the phase velocity of the jet wave at a point very near the aperture is four-thirds times the velocity of the jet particles at that point, and that for points not closely adjacent to the aperture the particle and phase velocities are for practical purposes equal.)

A first version of this paper was written in late 1971 and early 1972. The present version was written in 1974. Revisions were made in 1999 and 2000. The original paper stated only casually that the oscillation would be considered as a perturbation to the motion of the jet. A major purpose of the recent revisions was to emphasize that the theory being presented was a perturbation theory, and to show that this approach was fully justified. Text to that purpose has been added. No equations and no conclusions were changed. One new equation, equation 7,

was introduced. The literature since early 1974 has not been reviewed. A detailed account of the theory has not been previously published.

In mid 1971 the author got sufficiently interested in the flute and organ pipe to want to know how they worked (the author has a Ph.D. in Physics received in 1952 from the University of California in Berkeley where he worked under the direction of Professors Edwin M. MacMillan (a Nobel prize winner in Physics) and Wolfgang K. H. Panofsky in the late 1940s and early 1950s). A survey of the acoustics literature established that much is known about the operation of these musical instruments and that many books and thousands of papers had been devoted to them and people knew how to build excellent instruments, but that there were no theoretical papers which gave an adequate theory of their operation. It seemed to be established however that the flute and organ pipe were related to and somehow dependent upon the acoustic edgetone oscillator, which seems to be a drastically simplified version of these musical instruments incorporating some of their features but lacking the resonant acoustic air column which controls the frequency they produce.

A search of the literature devoted to the edgetone oscillator established that for this oscillator there were relatively few papers giving basic experimental data on its operation, the best of which were the papers of Brown (published in 1937), and Carriere (published in 1925), both referenced in the paper being discussed here. There were literally hundreds of theoretical papers by many authors trying to explain the operation of the edgetone oscillator and particularly the results of Brown.

Brown's experimental data is excellent, but there are serious problems in his presentation and interpretation of his data which were not recognized by these theorists. Dr. Alan Powell, to be mentioned again later, was and has been perhaps the most prominent of these, having published many letters and papers starting about 1953 devoted mostly to the elaboration of an approach and equation he had proposed in 1953 in his attempt to fit an empirical equation to Brown's data. Dr. Powell had accepted as correct Brown's faulty interpretation of his data. Although Powell did develop a large following among others who adopted his approach and equation in their own attempts to explain edgetones, his approach has never been successfully applied to predict Brown's basic data.

For some reason the equally important experimental results of Carriere have been almost universally ignored by theorists. None of their many papers has offered a theory that can be applied to predict Brown's experimental data or Carriere's.

In addition to the many papers published in the effort to explain the production of

edgetones, edgetones are briefly mentioned in many books devoted to explaining how common musical instruments, including the flute and organ flue pipe, work. Most of these books and papers and the discussions of the flute or organ pipe at least mention and some give great stress to the assertion that the phase velocity of the jet wave in the aperture to edge gap of the edgetone oscillator or of these musical instruments is only a small fraction of the velocity of the jet particles whose motion gives rise to the oscillations. A review of the literature shows that what the assertion or statement means is never clearly stated. A definition is necessary because the jet particles do not have a constant velocity all across the gap from jet aperture to edge in either the edgetone oscillator or the flute and organ pipe, the jet particles slow down. This slowing is very important for the edgetone oscillator, although of lesser importance usually for the flute and organ pipe. The simplest assumption is that the statement means that the jet particle velocity and the phase velocity of the jet wave in the gap are both constant across the gap and that the phase velocity is much less than the jet particle velocity. Another possible assumption is that the statement means that the phase velocity of the jet wave at a given position in the gap is a small fraction of the jet particle velocity at that same position in the gap, with nothing implied about the magnitude of the velocities at a given position. But actually the exact meaning of the statement is never expressed.

Apparently the first of these two options just given was tacitly assumed but not explicitly expressed in all prior attempts to explain Brown's results, and this was in fact tacitly assumed but not stated by Brown himself in presenting his results. But neither version of the interpretation of the statement is correct. Except very near the jet aperture the phase velocity of the jet wave at a point in the gap is almost equal to the jet particle velocity at that same point, this is established empirically by Schlichting's equations for the slowing of the jet particles and Brown's experimental data for the phase velocity of the jet wave near the edge, without the necessity of introducing any theory of the wave motion in the gap. Therefore neither the jet particle velocity nor the phase velocity of the jet wave is constant across across the gap, both velocities are for practical purposes equal to each other at a given point in the gap and both decrease as the gap is traversed. The major error in Dr. Powell's and other theorists' work has been their uncritical acceptance of the general belief, based upon their acceptance of Brown's erroneous interpretation of his data, that the phase velocity of the jet wave in the aperture to edge gap is a small fraction of the jet particle velocity in the aperture to edge gap. This belief resulted from their failure to to give an unambiguous definition of this statement.

It is evident that a clear understanding of the operation of the edgetone oscillator will not be achieved until the shibboleth about the phase velocity ingrained into

the minds of those studying the edgetone oscillator for the past seventy years and adopted in almost every paper or book discussing this oscillator is eliminated from the phenomenology assumed for the oscillator. This author is not aware any book or paper, other than his own, discussing the edgtone oscillator that presents a correct discussion of the relation between the phase velocity of the jet wave in the aperture to edge gap and the velocity of the jet particles in the gap. In general these books and papers exhibit no recognition even that the jet particles do not maintain their initial velocity just out of the aperture as they cross the gap.

To summarize, the phase velocity of the jet wave in the edgetone oscillator and in musical instruments such as the flute and organ flue pipe is not a small fraction of the jet particle velocity, as stated to be the fact in most texts and papers dealing with this oscillator and these musical instruments. To slightly adapt here a short but very appropriate quote from T. Needham written about a different subject but for a similar situation (T. Needham, Visual Complex Analysis, Oxford University Press, 1997, p. 386): *"We have made a fallacy of an assertion that is to be found in most texts. Perhaps the mere frequency with which this myth has been reiterated goes some way to explaining how it has acquired the status of fact."*

Brown was able to determine phase velocities only for stages two and higher and for these stages only just before the edge. Actually Brown measured the wavelength in the jet wave just before the edge. Multiplying this wavelength by the frequency observed gave the phase velocity of the jet wave just before the edge. The phase velocities Brown measured just before the edge were a small fraction of the jet particle velocities just out of the jet aperture. So Brown erroneously made the sweeping conclusion that the phase velocity of the jet wave was a small fraction of the jet particle velocity. Brown should have recognized that the jet particles slow down in crossing the aperture to edge gap and should have compared the phase velocity he measured near the edge to the jet particle velocity also near the edge, which would have been a more natural comparison. The phase velocities Brown had measured just before the edge were actually for practical purposes just the velocities to which the jet particles had slowed just before the edge but Brown had not recognized this, nor was this recognized by Powell and the many others trying to explain Brown's data, who had accepted as correct the main features of Brown's erroneous interpretation of his experimental data. The near equality of these two velocities is demonstrated in the paper under discussion here.

Since the phase velocity Brown found just before the edge was for practical purposes equal to the velocity of the jet particles just before the edge, it is strongly suggested that the phase velocity of the jet wave varies with position in the aperture to edge gap since the velocity of the jet particles varies with position in

the gap, the jet particles slow down as they cross the gap. This automatically means that the wavelength of the jet wave varies with position in the gap. But implicit in Brown's treatment and interpretation of his experimental data are the tacit assumptions that the wavelength is constant across the gap, and that the phase velocity and jet particle velocity are also constant across this gap.

Furthermore Brown had no experimental values whatever for wavelengths or phase velocities in stage one oscillations. But he inexplicably included values for stage one in his table of experimental values. A careful reading of his paper reveals that his values for stage one were guesses he had made. These entries should not have been included in his table but they were apparently accepted as genuine experimental data by later theorists. No one had pointed out that the entries were invalid.

Once it is acknowledged that the jet particles do slow down quickly as they cross the aperture to edge gap and the error about the jet wave phase velocity is eliminated from our mental picture of the phenomenology of the edgetone oscillator, it becomes rather an easy task to develop a theory of the oscillator, as demonstrated in my paper. The paper does predict every essential feature known about edgetone production then and now, including the prediction of the phase velocities Brown measured, which no one else as yet has been able to do. The basic data about edgetone production is best presented still in the papers of Brown and Carriere, and any acceptable theory of edgetones must predict their results.

It seemed to the author that surely this edgetone oscillator was just a simple feedback oscillator that could be explained in the same way that a multitude of electronic oscillators have been explained. My initial guess was that it might be an acoustic analogue of an electronic transit-time oscillator which is governed by the equation $fT = k$ where $f$ is the frequency of the oscillator, $T$ is the time required by electrons to cross a gap, and $k$ is a constant that depends upon the mode of oscillation. This electronic transit time oscillator has been carefully analyzed by Dietrich Marcuse (Dietrich Marcuse; Engineering Quantum Electrodynamics; Harcourt,Brace & World, Inc.; 1970; pp. 135-140). For the edgetone oscillator I thought $T$ would be the transit time of a jet particle from the jet aperture to the edge. A search of the literature showed that it would be easy to calculate this transit time for the edgetone oscillator but that apparently no one had ever investigated the possibility that the edgetone oscillator might be a simple transit time oscillator.

This was before the availability of small handheld electronic calculators, and the calculation of $T$ by hand for Brown's experiments although simple would be tedious, so I set up the equations now given in my paper for the calculation of $T$

with b, X, and v(0) as inputs and gave the equations and Brown's experimental data to a friend who had access to the big mainframe computer at the company where I worked and asked him to calculate the fT product for me for every entry in Brown's table of experimental data giving f as a function of the parameters b, X, v(0), and stage number in Brown's experiment. In about two weeks he provided me the data displayed in Table 1 of my paper "The theory of the edgetone oscillator." This table established empirically beyond any doubt that my guess was correct, the edgetone oscillator was indeed a transit time oscillator, governed by the simple equation fT = k where each operating mode of the oscillator has its own unique value of the parameter k. These computer calculations also established from the experimental data that the values of the constant k for the oscillator stages 1, 2, 3, 4, etc., would be very nearly k = 0.500, 1.250, 2.250, 3.250, etc. This equation obviously requires that the calculation of the transit time T take account of the slowing of the jet particles in crossing the gap from the jet aperture to the edge in those cases where the distance X is so great that a significant slowing of the jet particles has occurred. In Brown's experiments in many of his reported cases it can be shown that the jet particles had slowed to less than forty percent of their velocity just out of the jet aperture by the time T at which they reached the edge. Nevertheless Brown in his treatment and interpretation of his data had ignored the slowing of the jet particles, in effect tacitly assuming that the jet particles did not slow down.

By the time I had received the data of Table 1, I had developed equations 16 through 20 of my paper but had not yet seen how to interpret them to predict the fT values that would result in oscillations, although I was convinced that they could be so interpreted. I also saw that equation 8 of the paper for all practical purposes predicted as the particle velocities to which the jet particles had slowed at the edge distance just the phase velocities which Brown had measured for the jet wave just before the edge. Then I saw that the simple interpretation of equation 19 now offered in my paper would predict almost exactly the empirical fT values that had been developed from Brown's data and summarized as equation 15 of my paper. My equations also predicted that phase velocities measured near the edge should for all practical purposes equal the velocity to which the jet particles had slowed near the edge, and indeed the experimental data of Brown's indicated that this was true.

I was satisfied that I had now explained the basic features of the operation of the edgetone oscillator and had provided a means of calculating from theory without introducing any empirical factors everything that Brown had observed in his experiments. I could also explain what Carriere had observed, if one accepts the use of the empirical numerical factors commonly found necessary to correct the jet velocities calculated from Bernoulli's equation to the jet velocities actually

measured in experiments (Carriere gave the blowing pressure in his experiments rather than the initial velocity of the jet particles.) It had also turned out that no theory of the operation of the edgetone oscillator was really necessary. Brown's basic experimental data and the material in the literature on the slowing of the particles in fluid jets after they leave the jet aperture were all that was required in order to show that the edgetone oscillator was a transit-time oscillator governed by the equation $fT = k$ where k depends upon the oscillation stage or mode.

I wrote all of this up in a paper very much shorter than the present expanded paper, including every equation 1 through 25 of the present paper but the introductory equation 7 which I put in later, and including Tables 1, 2, 3, 4, and 5 of the present paper. These tables show that my theory predicts with great accuracy everything that Brown and Carriere found in their experiments. I submitted the paper in late 1971 or very early in 1972 to the Journal of the Acoustical Society of America for consideration for publication. The Journal sent the paper to an employee of a Naval Research Laboratory in San Diego for a decision on publication, who sent it to an employee of another Naval Research Laboratory who was (presumably) an expert on the edgetone oscillator, actually to the Dr. Alan Powell mentioned above who was then head of the Navy's David Taylor Model Testing Basin in the Washington, D.C., area, and later in 1990-1991 president of the Acoustical Society of America, for review. Dr. Powell is evidently an accomplished administrator, although it is the opinion of the present author, that it has not been demonstrated that Dr. Powell is a sound scientist, his many years of effort devoted to attempts to explain the operation of the edgetone oscillator have not led to any significant increase in our understanding of the oscillator, and in fact the present author believes that his influence has been a hindrance to the development in the interested community of a good understanding of this oscillator.

Dr. Powell objected to the paper and recommended in the strongest terms that it not be published, stating that it was in conflict with the fact established in Brown's experiments that the phase velocity of the jet wave was only a small fraction of the jet particle velocity and not equal to or greater than the jet particle velocity as my paper would indicate (Apparently Dr. Powell had no objections to my being informed that he was the reviewer. I was informed of the identity of the reviewer and actually had the opportunity not only to see his comments but also later to speak to him.) It did no good to point out to Dr. Powell and the editor in San Diego that Brown had made egregious and incorrect assumptions in interpreting his data that made his conclusion about the phase velocity invalid, and that any paper on edgetones that predicted with great accuracy every aspect of Brown's and Carriere's data in exquisite detail, including the prediction of Brown's measured values of phase velocity, without the necessity of any empirical factor introduced

to force a fit to the data, ought to be published. My personal view at that time and now is that as soon as Brown's untenable and obviously incorrect assumptions had been pointed out, it should have been obvious to any qualified physicist that Brown's interpretation of his admittedly very good experimental data was wrong.

I spent almost two years until the end of 1973 trying to convince Dr. Powell and the San Diego editor that every conclusion of my paper about the edgetone oscillator depended only upon the validity of Brown's basic experimental data and the validity of the well known equations for the slowing of a turbulent jet, and was independent of the theory I offered for the jet wave in the gap. If Brown's data and Schlichting's equations for jet slowing were correct, and there was no reason to doubt their correctness, then every conclusion of mine about the edgetone oscillator was correct on a purely empirical basis, with no theory involved although I believed my theory was correct. I also told the editor in San Diego that I thought Dr. Powell would be embarrassed by and uncomfortable with my paper because it made nonsense of almost everything Dr. Powell had published on edgetones in the previous twenty years, and there was a lot, so it was clear that he might dislike my paper, even though it accomplished everything that Dr. Powell himself had been trying to do since at least his paper of 1953, that is, it presented a method to predict everything that Brown saw. But Dr. Powell's objections to the publication of the paper prevailed. Finally after a delay of almost two years, near the end of 1973, the San Diego editor told me he would not accept the paper.

Dr. Powell was a strong supporter of the statement that the phase velocity of the jet wave was a small fraction of the jet particle velocity, ill defined though that statement may be. The remarks in my present paper about the inadequacy of Dr. Powell's work on edgetones were added only after my paper had been rejected. I wanted to make clear what the conflicting theories were, what objections to my paper were made that resulted in its rejection, and why those objections were ill-founded. I am still unable to understand why a reputable scientist of Dr. Powell's standing would object to the publication of a paper that predicted with great accuracy every thing that Brown and Carriere had observed about the edgetone oscillator, as demonstrated by tables 2 through 5 of my paper and by the comparison of the phase velocities Brown observed near the edge to the jet particle velocities near the edge, when the goal of every scientist working on edgetones was to make those predictions. Dr. Powell himself in the many letters and papers he had published about edgetones was trying to make those predictions but had never been able to accomplish it.

Short papers presented at meetings of the Acoustical Society of America were not subject to review and could be presented by any qualified member of the Physical Society, so to get the work on record I gave a short summary of the work at the

Los Angeles meeting of the Acoustical Society of America held 30 October - 2 November, 1973. An abstract of the presentation as indicated above was published in 1974. The most important equations of this paper are included in the abstract [W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)].

Since this work was done as a hobby and had nothing to do with what I did for a living, and there appeared to be an insuperable obstacle (i.e., the objections of Dr. Powell) to getting it published in JASA, and also because it was occupying a large portion of my spare time, I decided to drop the matter. However I did prepare a greatly expanded version of the paper in early 1974 which presented the theory in more detail and pointed out in considerable detail my findings about the phase velocity of the jet wave, which was the area where I think Dr. Powell was most seriously in error in making his recommendation and the editor in accepting the recommendation.

I then put the paper aside for almost twenty five years, during which time I made no effort to follow the literature on edgetones. During those years apparently almost no progress was made in explaining the operation of the edgetone oscillator although many papers were still being published. There was still no published theory or even an empirical procedure that could predict the experimental data of Brown and of Carriere.

With the astounding development of the Internet in the 1990s, there was no bar to my publishing the paper myself on the Internet. So I decided after I got myself a modern computer and access to the Internet, that there were surely some people who would be interested in the paper. I made it available on the Internet in early 1997. A search of the Internet with the Google search engine using either "edgetones" or "organ flue pipes" as the search topic should turn up my paper. I have made a few minor changes of wording in editing the paper for the Internet and I added later the section of the paper treating the data of Dr. Suematsu on the edgetone, but the paper is largely as I wrote it in 1974.

It is still my opinion as a professional physicist, although now retired, that the Journal of the Acoustical Society of America was seriously wrong to reject the paper and Dr. Powell was certainly wrong to recommend the rejection and the editor in San Diego was wrong to accept Dr. Powell's recommendation. The paper does predict everything that either Brown or Carriere saw in their experiments on the edgetone oscillator, there are no other good summaries of the facts of edgetone oscillator operation, and ordinarily such a paper should be published. Indeed, looking at the extremely good accuracy with which the experimental results of both Brown and Carriere were predicted as shown in Tables 2 through 5 of the paper, I find it almost incredible that two presumably competent scientists would

decide that the paper was nonsense and should not be published. Dr. Powell's professed reason for his recommendation was clearly wrong since it was easily demonstrated that Brown's experiments did not show that the phase velocity of the jet wave was only a small fraction of the jet particle velocity.

It is surprising that more than sixty years after the publication of the papers of Brown and Carriere and the publication of literally hundreds of experimental and theoretical papers dealing with the edgetone oscillator, the papers of Brown and Carriere are still the best papers presenting experimental information on the fundamental properties of edgetone oscillators. Just beware of Brown's interpretation of his data. If you need quantitative information on how the frequency of the oscillator depends upon the physical parameters of the jet edge system and the initial velocity of the jet particles you still need to go to the papers of Brown and Carriere. Their basic experiments have not been repeated. Indeed it seems that mainstream physicists ceased to have much interest in these acoustic oscillators more than sixty years ago which I think is perhaps a major reason their operation has remained unexplained in the published literature. Another reason of course is that almost everyone working to explain the edgetone oscillator, such as Dr. Powell, seems imbued with the conviction that Brown's interpretation of his data is correct. Almost every book or paper the author has found that discusses the edgetone oscillator or the operation of flutes or organ pipes has the erroneous statement that the phase velocity of the jet wave is only a small fraction of the jet particle velocity; adding to the difficulty is that no exact meaning for this statement is ever given. Mainline physicists went from the discovery of the quantum to quantum mechanics in twenty five years, that is in one sixth of the one hundred fifty years since the discovery of the edgetone oscillator, but the edgetone oscillator is still not explained in the published scientific literature, except for the present paper under discussion which is found only on the Internet. The explanation of the operation of edgetone oscillator turns out to be simple. Its behaviour is quite analogous to the that of conceptually similar electronic oscillators.

Return to The Home Page of Wade Blocker.

Comments will be appreciated.

wblocker@nmol.com

# THE HOME PAGE OF WADE BLOCKER

Comments will be welcomed.
wblocker@nmol.com

---

This page contains links to technical papers by Wade Blocker dealing with the application of the ordinary theory of electronic feedback oscillators to acoustic feedback oscillators. It is suggested that the papers be read in the order listed. To select a paper, left click on the the colored link.

---

## The theory of the edgetone oscillator

Size of file about 140 KB

---

## A theoretical model for the edgetone oscillator

Size of file about 9 KB

---

## The Development and History

## of

## "The Theory of the Edgetone Oscillator"

Size of file about 22 KB

---

## Distributed Circuit Oscillators

### Electronic Feedback Oscillators

**and**

**Acoustic Feedback Oscillators**

**Compared and Contrasted**

Size of file about 14 KB

---

## On the Pitch of Organ Pipes

**An Analysis of Data of Lord Rayleigh's**

Size of file about 30 KB

---

## The Edgetone Oscillator as The Exciting Agent

## for

## The Organ Flue Pipe

Size of file about 14 KB

---

## A THEORETICAL PREDICTION

### OF THE

# WEISENBERGER SCALING LAWS

### FOR THE

## ORGAN FLUE PIPE

Size of file about 16 KB

---

Comments will be welcomed.
wblocker@nmol.com

# The Theory Of The Edgetone Oscillator

## Initially Written 1974; Revised 1999 and 2000

Last Revision: Oct 6, 2000; minor notes added March, 2001


### Wade Blocker

wblocker@nmol.com


### Table of Contents



Return to The Home Page of Wade Blocker.

### Abstract

A theory of the edgetone oscillator is presented which predicts all critical experimental data. This is the first theory to accomplish this. The edgetone oscillator is proved to be a transit time oscillator closely analogous to electronic oscillators. No empirical elements are required in the theory in order to fit the experimental data.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

### Introduction

Although such musical instruments as the panpipe, recorder, flute, organ flue pipe, and common whistle had been known for millennia, it was only discovered in 1854 by Sondhaus (Ref. 1) that the resonant acoustic column or cavity associated with these instruments is not necessary for a tone to be produced. A tone is

produced when a jet of fluid from an aperture is blown against an edge. Since the discovery of these edgetones an extensive literature has accumulated, but past efforts to explain this edgetone oscillator have been unsuccessful. In 1940 Lenihan and Richardson (Ref. 2) wrote *"The problem of edge tones is one which continues to form a battle-ground for rival theories, though a complete solution seems as far off as ever."* This statement has remained a challenge for theorists to this date. No prior theory of edgetones has been able to predict the data of basic experiments. In this paper it is assumed that the edgetone acoustic oscillator is just another feedback oscillator that can be explained in a manner closely analogous to how ordinary electronic feedback oscillators are explained. The assumption that the edgetone oscillator is a transit time oscillator very much like electronic transit time oscillators leads to a theory in almost exact numerical agreement with all critical experimental data, with no empirical or *ad hoc* elements introduced to force a fit to the data.

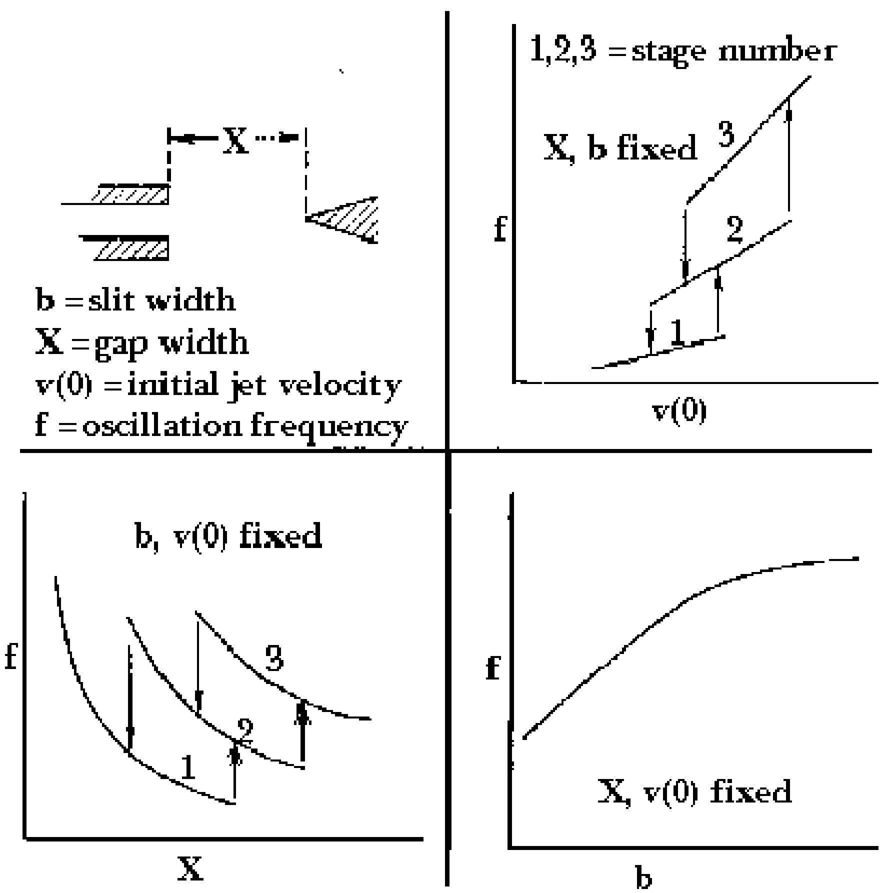
Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Edgetone Phenomenology

The basic phenomenology of edgetones seems simple. A tone is produced when a jet of fluid from an aperture is blown against an edge. Oscillations occur in both liquids and gases. Figure 1 shows the experimental facts.

# Figure 1




The upper left quadrant of Figure 1 defines the significant parameters in edgetone production. The jet and

aperture-edge system are immersed in a surrounding fluid. Denote the aperture slit width (that is, the distance between the upper and lower boundaries of the slit in Figure 1, or equivalently the thickness of the jet immediately out of the aperture) by b, the aperture to edge gap distance by X, and the velocity of the jet particles immediately out of the jet aperture by v(0). These parameters b, X, and v(0) fix the frequency f of oscillation, and it is also necessary to specify the oscillation mode or stage number. In this drawing take the origin of coordinates at the center of the aperture's exit, with the positive direction of the x-axis extending to the right through the point of the edge, the positive direction of the y-axis extending upward in the plane of the drawing perpendicular to the x-axis, and the positive direction of the z-axis extending outward perpendicular to both x and y axes toward the reader. The x-coordinate of the edge point is X. The edge and the plane jet of fluid emitted from the aperture in the positive x-direction will be considered infinite in both z-directions, thereby reducing the problem to be analyzed to two dimensions. The x-z plane divides space into two halfspaces, an upper and a lower.

The initial velocity of the jet particles immediately out of the jet aperture is v(0), and the velocity of the jet particles at the position x in the aperture to edge gap is v(x). At the edge position X the jet particles have slowed to the velocity v(X). The failure to give attention to the slowing of the jet particles is almost surely the major reason for the previous failures to develop an adequate theory of the edgetone oscillator.

The upper right quadrant of Figure 1 shows the effects of changing v(0) for fixed b and X. As the velocity increases from zero, oscillations begin at some point and the edgetone is produced. The oscillation frequency then increases with velocity along the lowest line indicated by the numeral 1. At some velocity the frequency jumps upward and then increases along line 2. With a further velocity increase the frequency jumps to line 3. As many as five or six jumps have been seen. Beginning with the lowest line, these modes of oscillation are called stage 1, stage 2, stage 3, etc. With a velocity decrease the frequency decreases along one of the lines and downward jumps back to stage 1 occur. The upward and downward jumps do not necessarily occur at the same points so this oscillator shows hysteresis just as most oscillators do. The lines are straight, and if extended go through or very near the origin.

The lower left quadrant of Figure 1 shows the effect of changing X for fixed b and v(0). As the gap width from aperture to edge increases, oscillations begin at some point. The frequency then drops as the gap width increases. There are frequency jumps as the gap is increased and again hysteresis effects are seen. The stages identified are the same as those seen when the velocity was varied. Most experimenters have found the frequency in a fixed stage to vary inversely as the first power of the gap width, but some experiments have been done in which the frequency varied inversely as the three-halves power of the gap width. These latter experiments have been mostly ignored by theorists attempting to explain edgetones, which is another reason for previous failures to explain edgetones.

The lower right quadrant of Figure 1 shows the effect of changing the slit width b for fixed v(0) and X. As b increases, oscillations begin at some point. The frequency then increases as the slit width increases and appears to approach a limit. Presumably jumps between stages might occur but they have not been reported. Most papers on edgetones do not identify the slit width b as an important parameter of these oscillators but give attention only to initial jet particle velocity v(0) and aperture to edge gap width X. But the slit width b is as important as the gap width X in determining the edgetone frequency.

Most writers on edgetones would add to the experimental facts just given that the jet disturbance in the gap when oscillations are occurring is characterized by a phase velocity which is about half the jet particle velocity, both velocities being tacitly assumed constant across the aperture to edge gap. This is an inference from certain theoretical and experimental work. The present paper shows that this inference is incorrect. Its general acceptance by theorists is another major reason for prior failures to develop an adequate theory of edgetone production.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Past Experiments and Theories

The number of experimental and theoretical papers written on edgetones is very large, certainly in the high hundreds. Only a few of the most important will be reviewed briefly here.

For the first oscillation stage, Koenig in 1912 (Ref. 3) proposed the empirical equation

$$f = v(0)/2X \qquad (1)$$

He did not explain why the factor 2 in the denominator was necessary to fit the experimental data.

Schmidtke in 1919 (Ref. 4) thought that the different frequencies observed in different stages should be harmonically related. Later more accurate experiments showed this to be incorrect. Schmidtke extended Koenig's equation as

$$f = nv(0)/2X \qquad (2)$$

where n was the stage number.

Krueger in 1920 (Ref. 5) proposed that the factor 2 in the denominator of equations 1 and 2 was properly associated with the jet particle velocity v(0) rather than with the aperture to edge distance X and indicated that the phase velocity of the jet disturbance in the aperture to edge gap was only half the jet particle velocity. This proposal was later almost universally accepted by those attempting to explain edgetones. It did give a plausible reason for the otherwise unexplained factor 2. However, the proposal will turn out to be wrong. His form of the frequency equation would be

$$f = n[v(0)/2]/X \qquad (3)$$

CarriÃ¨re in 1925 (Ref. 6) published experimental results showing the variation of the edgetone frequency with slit width b, with other parameters fixed. This is the only data the author is familiar with that gives frequency as a function of slit width b and is the data on which the diagram of the lower right quadrant of Figure 1 is based. CarriÃ¨re's data show that the aperture slit width b is as important as the parameters X and v(0) in determining the edgetone frequency. Nevertheless, the influence of the parameter b on edgetone frequency f has been universally neglected by edgetone theorists. The parameter b does not appear in any equation they propose for the edgetone frequency.

CarriÃ¨re also gave experimental results in which the frequency varied inversely as the three-halves power of the aperture to edge distance X. Both major experimental findings just cited of CarriÃ¨re's on edgetone phenomenology have been verified by the later experimental findings of other writers on edgetones but the findings are still mostly ignored by theorists. However, his data will be extremely important in verifying the theory to be offered in this paper.

Brown in 1937 (Ref. 7) published an extensive and perhaps the best yet collection of experimental data. His given experimental data are excellent, and are of primary importance in verifying the theory of the present paper. Brown had made frequency measurements for more than one value of slit width b and states that the edgetone frequency decreases as the slit width decreases, but Brown gives detailed data in this paper for only one slit width. This is a major deficiency in Brown's paper, not necessarily a criticism of Brown since Brown makes no claim to have covered all aspects of edgetone phenomenology, but later writers on edgetones seem to have had the opinion that Brown's paper does cover every important aspect of edgetone phenomenology. It is a fact that, except for CarriÃ¨re, and an off-hand mention by Brown that slit width

affects the edgetone frequency, all writers on edgetones have failed to give serious consideration to the possible importance of the slit width b in determining the edgetone frequency.

Brown established that the frequencies in different stages are not harmonically related. His data give the edgetone frequency for a single slit width b as a function of initial jet velocity v(0) and gap width X. Although not stated by Brown, as pointed out later in this paper it can be deduced from Brown's data that for some conditions the edgetone frequency is inversely proportional to the three-halves power of the gap width X, thus Brown's data confirm this finding of CarriÃ¨re's. Brown's purely empirical equation for the frequency, slightly simplified, is

```
f = a v(0) / 2X                                                    (4)
```

where a = 1.0, 2.3, 3.8, 5.4 for stages 1, 2, 3, and 4.

Brown took photographs of smoke filled jets on which he was able to measure wavelengths of the jet wave near the edge. He was able to measure wavelengths only near the edge since only just before the edge was there enough detail in his photographs to allow a wavelength to be defined. These wavelengths multiplied by the frequency gave a phase velocity which was about one-half of the jet particle velocity at the aperture. Brown interpreted this to mean that the phase velocity of the jet wave was about one-half the jet particle velocity in general agreement with the supposition of Krueger. Implicit in this interpretation by Brown of his data are unstated assumptions which are incorrect. He tacitly assumed that the jet particles did not slow down, and that the wavelength and phase velocity which he determined just before the edge were the wavelength and phase velocity at all points in the gap. He gave values for the different stages of the gap width divided by his measured wavelengths, assuming this to be the number of waves in the oscillation of the jet in the gap. This number turned out to be approximately equal to or somewhat larger than the stage number, leading to the conclusion that the number of wavelengths in the gap was equal to or greater than the stage number. All of Brown's assumptions just mentioned are wrong and every conclusion of Brown's based on these assumptions is wrong. This will be discussed in detail later. Most later theorists attempting to explain edgetones relied heavily on Brown's data and his interpretation of that data. They overlooked the errors of interpretation which Brown made which this paper points out. In checking a theory against Brown's data, the excellent original data as recorded by Brown should be used and not the data as interpreted by Brown.

Brown's basic data are acknowledged to be excellent. But there are five major errors in Brown's interpretation of his data. (1) Implicit in Brown's interpretation of his data is the unstated assumption that the jet particle velocity at all positions in the aperture to edge gap is the same as the jet particle velocity just out of the aperture; in other words, Brown neglected the slowing of the jet particles as they crossed the aperture to edge gap. (2) Brown tacitly assumed (again an unstated assumption) that the wavelength in the jet disturbance in the aperture to edge gap at all positions in the gap was the same as the wavelength measured immediately adjacent to the edge. These two unstated and erroneous assumptions immediately led to three other errors. (3) Multiplying the wavelength measured just adjacent to the edge by the edgetone frequency, Brown obtained a phase velocity which he implicitly assumed to be constant and to be the phase velocity at all positions in the aperture to edge gap. (4) Taking the ratio of this phase velocity determined immediately adjacent to the edge to the jet particle velocity immediately out of the aperture, Brown concluded that the phase velocity of the jet wave at any position in the gap was a small and constant fraction of the jet particle velocity at that position in the gap, since he had already tacitly assumed that both velocities were constant and independent of position in the gap. If Brown had compared the phase velocity he measured at the edge to the actual jet particle velocity at the edge, not the jet particle velocity at the aperture, he would have found the two velocities to be for practical purposes equal to each other. (5) Since there was not enough detail in Brown's photographs of the jet wave in the gap to allow the number of wavelengths in the jet wave in the gap to be counted, Brown assumed the number of wavelengths was just the gap width divided by the wavelength he measured immediately adjacent to the edge. This resulted in Brown concluding that there were more wavelengths in the jet wave in the gap than was actually the case, since the wavelength becomes

smaller as the edge is approached since the jet particles are slowing down. All of these errors will be discussed in detail in later sections of this paper. Unfortunately not just Brown's experimental data but also Brown's interpretation of that data have been accepted by later theorists and made the basis of their attempts to explain edgetones. This is particularly true of the many papers (to be referenced later) of Powell on edgetones which will be discussed in some detail later. Particular attention is given to Powell's papers since Powell has been a very prolific writer on edgetones and his influence has been very great on many other writers who have adopted Powell's approach in their own attempts to explain edgetones. Unfortunately, as pointed out in the history of the present paper presented in another paper on this same web site, Dr. Powell has been a bar to the presentation in the published literature of this paper giving a contrary view of the interpretation of Brown's data, although Dr. Powell was aware that the contrary view enabled the prediction of every experimental fact about the edgetone oscillator that either Brown or Carriere had published.

The papers by CarriÃ¨re and Brown are presently the definitive papers that together best set forth the experimental facts of edgetone oscillations that are illustrated in our Figure 1. These are the authors whose data every theorist on edgetones should try to explain and predict. Brown's paper has received a great deal of theoretical attention, but no prior paper has presented a theory that can predict or explain Brown's experimental data. Also, the present author is not aware of any prior theoretical paper that gives attention to explaining CarriÃ¨re's results, although his results have not been challenged. CarriÃ¨re's data must be considered in order to have a complete picture of basic edgetone oscillator phenomenology.

Jones in 1943 (Ref. 8) published remarks that confirmed CarriÃ¨re's finding that for some conditions the frequency varied inversely as the three-halves power of the aperture to edge distance. Jones noticed that this occurred when the gap was wide and the jet was turbulent. Jones suggested a mechanism for the tone production similar to that adopted in the present paper. However Jones did not go beyond suggesting this mechanism to analyze quantitatively the consequences and did not produce a successful theory. Jones' remarks received little attention, although they offered the key for the solution of the edgetone problem.

Curle (Ref. 9) and Powell (Ref. 10) in 1953 both relied on the presentation and interpretation Brown gave of his data and, accepting Brown's conclusions about the number of wavelengths in the jet wave in the aperture to edge gap, each independently proposed for the edgetone oscillator frequency the purely empirical equation

$$f = [n + (1/4)]v(0)/2X \qquad (5)$$

where n is the stage number. This equation manages to conform to every erroneous feature of Brown's interpretation of his data pointed out above. It will be shown later in detail that Brown's experimental data, properly interpreted, offer no support for this equation, which must be recognized therefore as an unsupported conjecture. It cannot be used to predict the data of CarriÃ¨re and Brown. Nevertheless, Powell's paper has had a very great influence on later theorists, many of whom have taken Powell's paper and equation as a starting point in their own attempts to explain edgetones (Note added in 1999: for a recent example, see Ref. 20).

The frequency equations proposed were empirical. These equations do not give any attention to the inverse three-halves power dependency of frequency on gap width X seen by both CarriÃ¨re and Jones (and also deducible from Brown's data); nor to the strong dependency of frequency on aperture slit width b known to exist from the work of CarriÃ¨re, and confirmed by Brown although Brown gives no detailed data. The parameter b, the aperture slit width, is not a parameter of these equations; in effect, every theorist has ignored the dependence of edgetone frequency upon aperture slit width without offering any explanation for that ignoration. Theorists continued to treat the jet velocity as constant, despite the known fact that these jets slow down rapidly.

For fixed values of v(0) and X, these equations all predict that the ratio of the frequencies seen in two different stages is a function of the two stage numbers only, and is independent of the values of v(0) and X.

However there is no agreement between these equations as to the values of the ratios. These equations give no attention to how the parameter b might affect these frequency ratios. The true statement will turn out to be: "For fixed values of b, v(0), and X, the ratio of the frequencies seen in two different stages is a function of the two stage numbers only, and is independent of the values of b, v(0), and X".

Return to the Table of Contents

Return to The Home Page of Wade Blocker

## A Transit Time Oscillator, fT = k

There is another way of interpreting equation 1 than Krueger's. If X/v(0) is taken as an approximation to the time T required for the jet particles to go from the jet aperture to the edge, then equation 1 becomes fT = 0.50, so we see that it has taken one half-period of the oscillation for the jet particles to reach the edge. This suggests that a necessary phase condition on the oscillation of the jet wave in the gap is being satisfied in order that oscillations may occur. This suggests further that the edgetone oscillator may be a transit-time oscillator whose frequency is given by the equation fT = k where each oscillation stage or mode has its own value of k. The transit-time oscillator is a well known and quite common electronic oscillator.

Approximating the transit time T from aperture to edge by X/v(0) is valid for those cases in which the aperture to edge gap width X is not appreciably greater than the aperture slit width b, but we should not expect this to be a good approximation in those cases where the gap width X is very large compared to the slit width b. We should certainly not expect this to be a good approximation in those cases where the jet particles have lost a large fraction of their velocity by the time they have reached the edge distance X. It has already been stated that the jet particles slow down very rapidly in the aperture to edge gap, a fact that will be substantiated in detail later in this paper. When this slowing is taken into account properly, it will be easily demonstrated that the edgetone oscillator is in fact a simple transit-time oscillator governed by the same equation fT = k that applies to electronic transit-time oscillators.

To check the assumption that the edgetone oscillator is a transit time oscillator is simple, but since the present author has not been able to find any indications of such a check in prior published work on edgetones, it seems not to have been done previously. The only requirement is that we predict the transit time, which is easily accomplished. This oscillator has perhaps the simplest frequency equation possible. For a transit time oscillator, as we have already stated, the frequency equation is

$$fT = k \qquad (6)$$

where f is the frequency, T is the transit time, and k is a constant for a given stage or oscillation mode. A transit time oscillator is defined for the purposes of this paper as an oscillator whose frequency is predicted by this equation. There is a sequence of values of k, with each stage or oscillation mode having its own unique value of k. k is the number of whole periods of the oscillation in the transit time T plus a fraction which is any excess of fT over an integral number of periods. If T is approximated by T = X/v(0), this equation becomes very similar to the empirical equations adopted by previous theorists. The philosophies are very different however, since equation 6 demands a realistically determined transit time taking account of jet slowing. The slowing of the jet turns out to be critical.

With the assumption that the edgetone oscillator is a transit time feedback oscillator, analogous to electronic oscillators, explaining edgetones theoretically is reduced to two independent problems, predicting the fT product sequence, or values of k, as a function of stage number, and predicting the transit time T of a jet particle from aperture to edge. In making these predictions we will assume that the longitudinal motion (x-motion) and transverse motion (y-motion) of the jet particles are independent of each other. That is, we treat the oscillation as a perturbation to the motion of the jet. This procedure turns out to be successful.

The essence of the pertubational method of solving a problem is to take a somewhat similar problem with a known solution and to make small changes in the problem statement or the solution to adapt the known solution to the new problem. The special feature of the perturbational approach is that it takes advantage of what is already known about a somewhat similar physical problem. There are many possible variations in executing the method. This perturbation procedure is too well known in physics to require extensive justification. Perturbation methods are exceedingly common in modern theoretical physics. If a calculation is too difficult to be done exactly, a perturbation calculation frequently can give a very reliable result.

The only possibly justifiable objection to be raised against this procedure for our problem is that a perturbational calculation of the motion of the jet particles is not warranted and will not be successful. But that is a speculation that can only be resolved by testing the procedure. The validity of the perturbational calculation, or more properly its usefulness, in any particular case is to be judged by its success or failure, that is, by whether or not the calculation results in an explanation in agreement with established facts and available experimental data. This is the prime criterion by which the validity of a perturbation calculation is to be judged. The inevitable assumptions involved are considered justified if the procedure is successful, otherwise not.

It is possible for our problem to dispense with predicting the fT product sequence theoretically, and to determine the sequence empirically by simply calculating T using current theories of jet slowing, and then multiplying the experimental values of f in each stage by their corresponding T values. In this paper we will develop the fT product sequence by both methods independently, empirically using Brown's experimental data and the equations of jet slowing from the published literature, and theoretically using the theory being developed in this paper. The empirical approach will be shown first. The empirical result is that indeed each oscillation mode or stage has its own unique value fT. Therefore the edgetone oscillator is indeed a transit time oscillator. This will now be demonstrated.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Transit Time T of a Jet Particle

In determining the transit time of jet particle across the aperture to edge gap, we will start with Newton's fundamental law of mechanics:

```
"mass times acceleration equals force"
```

When applied to a unit volume element of a fluid, and assuming an incompressible fluid of unit density, this becomes

$$\mathbf{r}'' = \mathbf{f} + \mathbf{g} \qquad (7)$$

where **r**, **f**, and **g** are vector quantities. **r** is the vector giving the position of the volume element, and the total force on the element has been decomposed into body forces **f** and surface forces **g**. Prandtl and Tietjens (Ref. 11) present this equation as the fundamental equation of hydrodynamics and show how it leads to the Navier-Stokes equation. The reader is referred to their book for the details of this development.

We will adopt equation 7 as our fundamental equation. The density of the fluid is immaterial in our treatment of the problem so the density is not made a parameter of our problem. By assuming that the slit, jet, and edge are infinite in both z-directions, we eliminate any consideration of the z-direction from this equation of motion of a jet particle. In keeping with our perturbation calculation, we consider that the y-

motion of the jet particles when oscillations are occurring is a minor perturbation to the motion of the jet particles which has no effect on their x-motion and in turn is not affected by the x-motion. Our adoption of this perturbation method of treating the edgetone problem allows separate treatment of the x-motion and y-motion of the jet particles. We will consider the problem of the x-motion first. But we do not have to restate and solve equation 7 as it applies to the x-motion of the particles. This problem has already been solved for us in the published literature.

(Instead of using standard mathematical notation, I will write some following expressions as they would appear if written and intended to be executed in a Basic language program. In particular I will use * to indicate multiplication, and ^ to indicate exponentiation, and the standard rules for the order of evaluation are assumed.)

The transit time of a jet particle across the gap from aperture to edge when there are no oscillations will be taken as a sufficient approximation to the transit time when oscillations are occurring. The jet will be assumed turbulent. For narrow gaps this assumption even if wrong will cause no significant error in the calculated transit time and for wide gaps the jet is almost surely turbulent, as observed by Jones. What we are doing in this perturbational approach is to assume that the motion of the jet particles when oscillations are occurring is in most respects very similar to the motion of the jet particles in a non-oscillating turbulent jet, which latter problem has already been solved. The transit time of the non-oscillating turbulent jet can be calculated from the equations of jet motion for a turbulent two-dimensional free jet given in Schlichting's text on boundary layer theory (Ref. 12, pp. 605-607). For the velocity v(x) of a jet particle at the distance x from the jet aperture, we can deduce from Schlichting's discussion of the jet motion the equation

```
v(x) = v(0)/{[1 + (x/x0)]^(1/2)}                         (8)
```

Since equation 8 is just a restatement of Schlichting's results with a different choice for the origin of coordinates, we shall regard this equation as Schlichting's and attribute it to him. We have shifted the origin of coordinates from the hypothetical source point of the jet (assumed to be the origin of coordinates in Schlichting's equations) to the jet aperture where the velocity of the jet particles is known to be v(0). Equation 8 has also been specialized to the velocity of the jet at its centerline. x0 in equation 8 (Schlichting uses a different symbol for this, the letter small s) is defined from Schlichting's equations. In our equation 8, it is the distance before the jet aperture of the hypothetical source point of the jet. From the general equations given by Schlichting, it can be shown that

```
x0 = 3 s b/4 = 5.75 b                                   (9)
```

where s in our equation 9 (Schlichting uses the Greek letter small sigma) is an empirical constant with the value 7.67 which is given by Schlichting, and b is the slit width of the jet aperture. This x0 is defined by the condition that the momentum flux for the jet determined just out of the jet aperture is b*v(0)^2. This momentum flux is assumed by Schlichting to be conserved and to be the momentum flux at any position x reached by the jet particles. The fluid density cancels out as a factor in Schlichting's equations in deriving this result for x0. The jet behavior does not depend upon the density of the fluid, or upon whether the fluid is a liquid or a gas. This is actually our justification for disregarding the fluid density in stating equation 7.

Equation 8 shows that the slowing of the jet particles is a function of the ratio X/b of gap width X to slit width b. Therefore X and b are both of equal importance in determining the degree to which the jet particles have slowed in crossing the aperture to edge gap. Equation 8 shows that only when the gap width X is small compared to the parameter x0 can the velocity of the jet particles be considered constant all across the gap from aperture to edge.

Schlichting's equation (our equation 8) for the jet particle velocity applied to Brown's data shows that for a typical entry in Brown's data the velocity of the jet particles at the edge position is less than half of the initial velocity of the jet particles at the jet aperture. Previous attempts to explain edgetones have not given

attention to this extreme slowing of the jet particles.

From equation 8 the time increment dT required for a jet particle at position x to travel the distance increment dx is

```
dT = dx/v(x) = [dx/v(0)]*[1 + (x/x0)]^(1/2)
(10)
```

Integrating this equation with the initial condition that T = 0 when x = 0, we obtain for the time T(x) required for a jet particle to reach the distance x the result

```
T(x) = T0*{[1 + (x/x0)]^(3/2) - 1}                                   (11)
```

where

```
T0 = (2/3)*[x0 / v(0)]                                               (12)
```

When x is much smaller that x0, then equation 11 reduces to T(x) = x/v(0), so that the transit time T(X) can be adequately approximated by T(X) = X/v(0), but this approximation is not generally valid. For example, for many entries in Brown's experimental data the gap width X is many times larger than the parameter x0.

The equation for T(x) can be solved for x to give

```
x(T) = x0 * {[1 + (T/T0)] ^ (2/3) - 1}                               (13)
```

where x(T) is the distance traversed by a jet particle in the time T.

Equations 8 through 13 apply only to a plane jet of thickness b at the jet aperture. The equations applying to jets with a different geometrical configuration would be different, as is evident from Schlichting's discussion of this problem of jet slowing.

Equations 8 through 13 are either taken directly from Schlichting or follow directly from Schlichting's work. They are in essence Schlichting's equations. These equations are developed by Schlichting using hydrodynamics theory and are in full agreement with and violate no fundamental tenets of modern fluid dynamics and hydrodynamics. They involve no empirical elements or approximations in fluid dynamics theory introduced by the present author. These equations are not the result of a perturbational approach by Schlichting, but Schlichting has made the usual well established approximations of boundary layer theory. The value of the empirical constant *s* in equation 9 is given by Schlichting in his book, published many years before the development of the present paper. The comparison in Schlichting's text of theoretical and experimental results validates Schlichting's equations. These equations from Schlichting will be applied to determine an empirical fT product sequence. It is here that our procedure adopts the features of a perturbation calculation. We are assuming that the occurrence of oscillations would at most make only a small difference in the time required by the jet particles to cross the aperture to edge gap, or to reach any given position x in the gap. The assumption seems reasonable. The only legitimate test of our assumption will be whether or not the predictions we make about the behaviour of the edgetone oscillator agree with CarriÃ¨re's and Brown's experimental data.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Empirically Determined fT Product Sequence, fT = k

The most extensive compilation of experimental data is that of Brown. The data of Brown's Table 1 will be exhibited later in our Tables 2 and 3. For each entry in Brown's Table 1, using Brown's b, v(0), and X as inputs to our equation 11, the transit time T to the edge was calculated. This value of T multiplied by the frequency f that Brown observed for that case gives the results tabulated in our Table 1.

Table 1. The empirical fT Products for Brown's Table 1.

| Stage Number | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| f | fT, Experimental | | | |
| 20 | -- | 1.95 | 1.17 | 0.48 |
| 100 | 3.20 | 1.97 | 1.12 | 0.45 |
| 150 | 3.11 | 2.12 | 1.22 | 0.50 |
| 1200 | 3.31 | 2.24 | 1.24 | 0.47 |
| 2400 | 3.29 | 2.21 | 1.31 | 0.51 |
| Average | 3.23 | 2.10 | 1.21 | 0.48 |

The experimental values of fT in Table 1 are nearly constant for a given stage. We conclude that the edgetone oscillator is indeed a transit time oscillator. The near constancy of the experimental values of fT for a given stage establishes empirically that the edgetone oscillator is a transit time oscillator. Our Table 1 justifies the empirical fT products

fT = 0.50, 1.25, 2.25, 3.25 (14)

for the oscillation stages 1, 2, 3, 4

It is quite natural to postulate an extended sequence

fT = 0.50, 1.25, 2.25, 3.25, 4.25, 5.25, ... (15)

for the oscillation stages 1, 2, 3, 4, 5, 6, ...

We will now show that this empirical sequence can be explained by a simple extension of our theoretical procedure.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Theoretically Predicted fT Product Sequence, fT = k

It will be assumed that the edgetone oscillator is an ordinary feedback oscillator that can be explained in a manner exactly analogous to how electronic feedback oscillators are explained. A plausible feedback mechanism and feedback loop will be defined, and a search made for the phase condition on the feedback loop that results in the positive feedback that is necessary for continuous oscillations.

We have treated the x-motion of the jet particles taking full account of the principles of hydrodynamic theory by our adoption of Schlichting's equations for the x-motion of a turbulent jet. So we are left only with the problem of applying equation 7 to determine the y-motion of the jet particle.

The only body force **f** acting on the unit volume element is gravity which is negligible for this problem and can be neglected. If frictional forces in the y-direction are neglected, which is reasonable in our approximation because these forces are very small and are certainly small in comparison to the pressure difference across the jet, the only surface force **g** we have to consider in specializing equation 7 to the y-motion is the pressure drop across the jet in the y-direction.

The plane through aperture and edge divides all space into two halfspaces, an upper and a lower. Because of the mirror symmetry about the x-z plane, we assume, using the terminology of electronic oscillators, that this is a push-pull oscillator. This is a critical assumption of the present theory. It implies that the pressure fluctuations on the upper and lower sides of the jet should be 180 degrees out of phase when oscillations are occurring. The jet particles of necessity respond to the pressure difference on the two sides of the jet. We postulate that, driven by the alternating pressure difference between the two halfspaces which acts on the jet particles, the jet particles are directed in alternating puffs into the upper and lower halfspaces. This creates the alternating pressure difference between the two halfspaces, and this alternating pressure difference controls the y-component of the jet particle motion, thereby completing the feedback loop required for continuous oscillations to exist. This is exactly analogous to how the ordinary electronic push-pull oscillator is explained, with pressure variations in the acoustic oscillator being taken as the analog of voltage variations in the electronic oscillator. A similar mechanism has been suggested before [Jones (Ref. 8)], but was not followed to the point of producing a successful theory. With the oscillator identified as an acoustic feedback oscillator, and the feedback loop defined, we look for the phase condition on the feedback loop that reinforces the excitation and makes the oscillations self-sustaining.

Assume the pressure difference across the jet varies sinusoidally with time. If the pressure acting on the lower surface of the jet is +sin(wt), then the pressure acting on the upper surface is -sin(wt), since the symmetrical geometry of the jet-edge system dictates a push-pull oscillator. The net force acting to accelerate the jet particle in the positive y-direction is then 2*sin(wt), but we will neglect the amplitude factor 2 and take this force as just sin(wt). Since the aperture to edge gap distance in the usual case is a small fraction of the wavelength of the tone produced by the edgetone oscillator, we assume that this pressure difference has the same value all across the gap at any given instant of time. (The wavelength of the tone produced, c/f, where c is the velocity of sound in the fluid, should not be confused with the wavelength seen in the jet wave.) This sinusoidal pressure difference is the dominant influence on the transverse or y-motion of the jet particles.

Define w as 2*pi*f , t as clock time, e as the time a particular fluid particle is emitted from the aperture, and T as (t - e). T is thus the length of time that a particle emitted at the time e has been traveling since it was emitted from the aperture. For the usual Greek letter pi, I have substituted the word pi, and for the usual Greek letter omega I have substituted the letter w. The quantity f is the frequency of oscillation. The differential equation for the motion of the jet particles in the y-direction, a simplification of equation 7 fully consistent with Prandtl and Tietjen's discussion of this equation, q.v., is therefore

```
y'' = sin(wt)                                            (16)
```

with the initial conditions that y = y' = 0 when t = e or when T = 0. These initial conditions simply mean that the jet particles immediately out of the jet aperture have no displacement or velocity in the y-direction. Their velocity just out of the aperture of course in the x-direction is v(0). The primes indicate differentiation with respect to time. Any constant amplitude factor has been taken as unity in this equation since it would not affect the argument that will be made. Therefore the density of the fluid has not been considered in this equation. Any accretion or entrainment of mass by the jet as the jet particles traverse the aperture to edge

gap has been neglected. The justification of this neglect will be the agreement of theoretical predictions with experimental facts.

It is with this equation that we have introduced the effect of feedback into our theory. This equation gives the effect of the oscillations in pressure upon the motion of the jet particles thereby completing the feedback loop, since it is the motion of the particles that resulted in the oscillations in pressure.

The only features in the use of equation 16 to determine the y-motion of the jet particles that identifies our procedure as perturbational are that we are assuming that frictional forces acting on a jet particle in the y-direction are negligible and that the y-motion of the jet particles is small enough that the x-motion of the particles is not significantly altered by the occurrence of oscillations. For small enough amplitude of the oscillations this will certainly be the case. We are also assuming that the y-motion is not affected by the x-motion, which again should be true for small oscillations. So we have every reason to expect that our procedure, although perturbational in nature, should in the limit of small oscillations give an almost exact answer, and should for any oscillations give a very useful answer.

Integrating equation 16 with the given initial conditions, we obtain the solution

$$y(wt, we) = (wt - we)\cos(we) + \sin(we) - \sin(wt) \qquad (17)$$

Again any constant factor common to all terms on the right side of the equation has been ignored. From equation 17 we can derive other valid forms:

$$y(wT, we) = (wT)\cos(we) + \sin(we) - \sin(wT + we) \qquad (18)$$

$$y(wt, wT) = (wT)\cos(wt - wT) + \sin(wt - wT) - \sin(wt) \qquad (19)$$

$$y(wt, wT) = (wT)\cos(wt - wT) - 2\sin(wT/2)\cos[wt - (wT/2)] \qquad (20)$$

It should be noted the parameter T is just another way of specifying a position x in the gap (see equation 13), so that as T increases x has also increased. Our equations for the x- and y-displacement of a jet particle are both given as functions of the parameter T, that is, we are defining the path of the jet particles and the resulting jet waves parametrically.

There are two frequently used and equally valid ways of dealing with wave motions in material bodies. One way is to define and work with the wave description from the start in handling the problem, and a second is to start with the motions of the individual particles making up the wave. The description in this paper of the motion of the jet starts with the individual fluid particles of the jet. It is the individual fluid particles that move. The particle motion is the reality. In a sense the wave and the motion of the wave are illusions. In our problem each particle travels its own path essentially unaffected by the motion of the other particles, but the overall view is that we have a wave carrying out its own motion. The wave is a composite made up of the particles, each particle of the wave having been emitted from the aperture at a different time. In many cases one mode of description of what takes place may have advantages over the other mode, but both descriptions have equal validity. In this paper it was found easier to deal with a description of the motion of the individual particles rather than deal with a description of the wave from the start, but the end result is a description of the wave. The equations for y(wt, wT) are descriptions of the wave. But it must be remembered that the x-part of the wave motion is described independently by equation 13 above for x(T). For any fixed instant of time t, the parametric equations for x(T) and y(wt, wT) as a function of T, together give a snapshot of the jet wave in the aperture to edge gap at the fixed time t.

This description of the jet wave is in agreement with every precept of fluid mechanics, subject only to the qualification that it is the result of a perturbational calculation which must be validated by comparing its

predictions with experimental data. The approximations made in doing the calculation however are quite reasonable and usual approximations so there are no obvious *a priori* reasons to doubt the results of this perturbational approach. But the conclusive test will be the comparison of predictions with experimental results.

The present theory does not predict or give consideration to the amount of energy lost by the jet particles to the acoustic oscillation, although it does take account that an interaction between the jet particles and the oscillations does occur which causes the oscillations to take place. Most of the energy loss of the jet particles is due to the entrainment of mass by the jet as it crosses the aperture to edge gap, and Schlichting's equations can be used to predict this loss. The theory as presented implicitly assumes that the efficiency of conversion of jet particle energy to oscillation energy while large enough to ensure the occurrence of oscillations is small enough that no significant additional velocity loss results because of the occurrence of oscillations, in conformity with a perturbation calculation which assumes that the interaction is small. However this does not affect the validity of our calculation of the oscillation frequency, but it does mean that we can say nothing about the amplitude of the oscillations or what conditions are necessary for oscillations to begin. Since the theory does predict edgetone frequencies quite well, the *a priori* assumption that the edgetone oscillator is a low efficiency oscillator is presumably justified.

At this point in our exposition it is probably not amiss to state that every approximation known to the author in this present treatment of the edgetone oscillator has now been identified and its possible importance assessed. This in marked contrast to past treatments of this oscillator. For example, in Brown's and Powell's treatment of Brown's data and in the usual treatment of the edgetone oscillator there are two major perhaps unrecognized, but certainly unacknowledged and hidden, simplifications of the physical situation.

The jet particles are assumed in past work to hold their velocity just out of the aperture all across the aperture to edge gap. This is an unstated assumption which ignores an actual velocity loss for Brown's usual case of more than 60 percent in crossing from the aperture to the edge. There is certainly no reasonable justification for ignoring a velocity loss of this magnitude in attempting to explain this oscillator. This velocity loss of more than 60 percent by the jet particles in crossing the aperture to edge gap means that the jet particles in these cases have also lost more than 60 percent of their initial kinetic energy before they reach the edge in crossing the gap. The velocity loss is the result of the jet gaining mass by accretion as the jet particles cross the aperture to edge gap with the momentum flux of the jet at any position in the gap the same as its value at the jet aperture. The conservation of the momentum flux is a key feature of Schlichting's equations. It follows immediately that the fraction of the initial kinetic energy lost by the jet is the same as the fraction of the initial velocity lost. This large energy loss strongly suggests that it is not hypothesized interactions of the jet wave at or with the aperture or edge that are the important factors in producing edgetones but that it is instead the very real interaction of the jet particles with the oscillating acoustic pressure field sin(wt) as they cross the gap from aperture to edge that gives rise to the oscillations. This latter hypothesis is adopted in this paper as the explanation of edgetone production. The important interactions occur not at either the aperture or edge but in the space between aperture and edge, since that is where most of the energy loss of the jet particles occurs. The position of the edge is very important, of course, since a boundary condition imposed on the jet wave at the edge will be the condition that determines the frequency of the jet-edge oscillations.

Additionally by calculating the number of wavelengths in the jet wave in the gap as "gap width divided by wavelength measured at the edge" it was tacitly assumed in past work that the wave length is constant across the gap and independent of position in the gap. There is neither theoretical or experimental justification for this assumption. The possible effects of this assumption in trying to explain edgetones were not assessed. Perhaps the existence of this assumption was indeed not recognized. But it causes a serious error in the phenomenology assumed for the oscillator. Theorists believed there were more wavelengths in the jet wave in the gap for each oscillation mode than actually existed. This mistaken belief led both Curle and Powell to postulate equation 5 above for the edgetone oscillator frequencies.

The two gross simplifications of the true physical situation just identified led to another false conclusion that the phase velocity of the jet wave was a small fraction of the jet particle velocity. These errors are probably the major reason for past failures to explain edgetones.

Equation 18 gives us the path in the aperture to edge gap of a particle identified by its time of emission e from the aperture. Equation 19 will give us the values of k in equation 6 which define the expected edgetone frequencies, and equation 20 will give us information about the phase velocity of the jet wave in the aperture to edge gap. Equations 19 and 20, coupled with Schlichting's equations from standard fluid dynamics theory for the slowing of a turbulent jet, will allow us to predict with remarkable accuracy the frequencies and phase velocities that Brown observed in his experiments. These predictions are made without introducing any empirical or *ad hoc* factors into the theory to force a fit to the experimental data.

The kinetic energy lost by the jet is the energy source for the edgetone oscillations. From a simple assumption concerning this energy loss a frequency condition can be derived which turns out to be in agreement with the experimental data. Assume that the instant of most rapid increase of pressure in a halfspace coincides with the instant of the greatest rate of loss of kinetic energy by the jet to that halfspace. The assumption will be justified by the success of the resulting theory. For a sinusoidal pressure variation, the instant of most rapid increase of pressure occurs as the pressure changes from negative to positive values and goes through zero. The instant of the greatest rate of loss of kinetic energy by the jet to a halfspace should be the instant of greatest mass commitment to the halfspace.

We see immediately that if the distance X from the jet aperture to the edge is such that the resulting transit time T of a jet particle from the aperture to the edge is such that fT = 0.500, a maximum net mass commitment to a halfspace results. The jet particles at the aperture always switch from one halfspace to the other in phase with the driving pressure and continue to flow into the halfspace for just a halfperiod, so a maximum net commitment of mass to the halfspace results if the distance to the edge is such that jet particles that are emitted from the aperture just as the pressure sin(wt) across the jet equals zero reach the edge distance one halfperiod after they are emitted from the aperture. This gives us a first value fT = 0.500 which we predict will result in oscillations.

But a greatest net commitment of mass to a given halfspace should occur also as the jet particles switch simultaneously at both the aperture and the edge out of that halfspace, instead of switching just at the aperture, as is the case for fT = 0.500. Jet particles just out of the aperture always switch from one halfspace to the other in phase with the driving pressure, that is, y(wt, wT) for the value wT equal to zero changes from negative to positive values as the driving pressure sin(wt) changes from negative to positive values, so we require that the same be true for the value of wT at the edge distance if oscillations are to occur. The driving pressure sin(wt) is zero and switches from negative to positive values when wt equals zero or a multiple of 2*pi, so y(wt, wT) at the edge must also be zero and switch from negative to positive values when wt equals zero or a multiple of 2*pi. This boundary condition which we have just imposed reduces equation 19 to an expression which we predict is a condition on wT at the edge that will result in oscillations.

```
wT = tan(wT)                                              (21)
```

This equation is easily solved with a programmable electronic calculator or computer, but it is also a well known equation whose solutions are tabulated in many texts or collections of mathematical data. The solutions meeting our criteria are the alternate zeroes of this equation. The solutions of interest lead to

```
fT =  1.230, 2.239, 3.242, 4.244, 5.245, ...              (22)
```

taking into account that

```
wT = 2*pi*fT.                          (23)
```

We have now made the prediction that the observed values of fT which will result in edgetone oscillations should occur in the sequence

```
fT = 0.500, 1.230, 2.239, 3.242, 4.244, 5.245, ...          (24)
```

for the stage numbers 1, 2, 3, 4, 5, 6, ... .

The boundary condition that determines the position of the edge when oscillations are occurring is that the mass committed to a half-space be a maximum when the acoustic driving pressure in that halfspace has its fastest rate of increase.

The theoretically predicted fT sequence of equation 24 should be compared with the experimentally (or empirically) established sequence of equation 15. The agreement is almost exact.

The sequence 22 gives the frequencies of best operation of the ordinary electronic transit time oscillator known as the klystron. This can be deduced from the analysis of the klystron oscillator given by Marcuse (Ref. 13). It is an elementary exercise to deduce from Marcuse's equation (4.3-14) on page 137 of his book that equation 21 above predicts the frequencies of best performance of the oscillator Marcuse designates as a Transit Time LC Oscillator. The procedure followed here in deriving equation 24 is quite similar in principle to the procedure used by Marcuse in deriving his equations for the klystron oscillator and indeed the resulting frequency equations can be shown to be identical except for the additional term fT = 0.500 in equation 24. Marcuse's derivation is certainly more sophisticated than the derivation here, and additionally his oscillator is not a push-pull oscillator such as the edgetone oscillator obviously is, and allowance has to be made for this difference. Although the details of our derivations differ on the surface, the fundamental idea of the two derivations is the same. In the klystron oscillator the electrons in interacting with the oscillating electric field either lose energy to the electric field, in which case oscillations continue, or gain energy from the field in which case oscillations die away. In the edgetone oscillator the jet particles in interacting with the oscillating acoustic field sin(wt) in the aperture to edge gap either lose energy to the acoustic field, in which case oscillations occur, or gain energy from the field in which case oscillations die away. Which of these two things happens depends upon the phase relation between the motion of the particles and the oscillating field. In the electronic oscillator depending upon the phase of the electric field oscillation when a particular electron enters the region where the electric field acts on it, the electron is either speeded up or slowed down; the result is a bunching of the electrons in the electron stream, with the final effect produced on the oscillator output depending upon the transit time of the electrons through the active field region. In the edgetone oscillator depending upon the phase of the acoustic field pressure oscillation when a particular jet particle enters the region where the acoustic field acts on it, the particle is either directed into the upper half-plane or the lower half-plane; the result is the production of a wave in the jet particle stream, with the final effect produced on the oscillator output depending upon the transit time of the jet particles through the active field region. In both oscillators the transit time of the particles must be such that the appropriate phase relation to result in the build up of the oscillating field, electric or acoustic, exists. This sets a condition that determines the oscillation frequency. I provisionally assumed the validity of my equation 19, and I then assumed a phase relation between the jet wave and the acoustic field which I thought was very probably necessary if oscillations were to occur, which gave me equation 21 which leads in straightforward fashion to equation 24, and I justify my assumptions by the end result, which is the complete agreement of predictions with Brown's data. Agreement with the experimental data is ultimately the only way any physical theory is justified and is the usual way of establishing the validity of a perturbational approach to a problem.

[Note added in March, 2001: Although the present theory recognizes the work done by the moving jet particles against the acoustic field to be the cause of the oscillations, it does not attempt to calculate the amplitude of the acoustic field oscillations that will result. This is not necessary to predict the edgetone

oscillator frequencies. However, such a calculation quite analogous to the calculation made by Marcuse for the klystron oscillator in deriving his equation (4.3-14) would appear, at least in principle, to be a straightforward addition to the present paper.]

The theoretical fT sequence of equation 24 for practical purposes differs only trivially from the empirical fT sequence of equation 15. Therefore our theory has predicted the experimentally established fT product sequence of equation 15. Equation 24 immediately accounts for the numerical factor of 2 required in the empirical equations 1 through 4 to fit the experimental data for stage one. This is evident upon approximating T by X/v(0) for stage one. This is not an adequate approximation for wide gaps, and consequently not usually adequate for the higher stages of oscillation, and perhaps not always adequate for stage one. By "wide gap" in the context of this paper is meant that the gap width X is very much greater than the slit width b. The slowing of the jet in crossing the gap is very important for wide gaps, amounting to more than one-half in the usual case as will be shown later. Equation 24 automatically makes provision for the effects of jet slowing.

Equation 24 gives the basic prediction of the present theory. This prediction is purely theoretical and has no empirical or *ad hoc* elements introduced to force a fit to the experimental data. Since this equation is in agreement with the experimental data, as shown by Table 1, it is established theoretically as well as empirically that the edgetone oscillator is a transit time oscillator. The transit time T is the controlling parameter determining edgetone frequency. The system parameters b, v(0), and X are of only incidental importance insofar as they determine T. All sets of these parameters giving the same value of T will produce the same frequencies f. Since the parameter T determines the frequency of the edgetone oscillator, the slowing of jet particles must be explicitly taken into account in any theory of the oscillator. The failure to do this is obviously the major failing of past theoretical efforts.

The transit time T as shown by equation 11 is a function of b, v(0), and X only; it is independent of the stage number of the oscillation occurring. Therefore equation 24 predicts that the ratio of the two frequencies seen in two different stages for the same values of b, v(0), and X in both stages is a function of the two stage numbers only and is independent of the particular values of b, v(0), and X. The combination of equations 11 and 24 of this paper predicts this fact and offers the explanation for it.

Brown's empirical equation for the edgetone frequency (our equation 4 above)was determined by a process that conforms to the condition on b, v(0), and X of the preceding paragraph. Brown's values for the empirical factor a in his equation for stages 2, 3, and 4 were found by successively multiplying the value a = 1.0 for stage 1 by the frequency ratios found for stage 2 to stage 1, stage 3 to stage 2, and stage 4 to stage 3, where each of these ratios was found for the same values of b, v(0), and X in the two stages for which the ratio was being determined. The values of v(0) and X were not the same in the determination of each ratio but were the same in determining the two frequencies involved in a ratio. The value of b was the same in all of Brown's frequency measurements. Brown's process is valid although it does compound the errors in the individual frequency ratios as they are multiplied together to determine the successive values for a. However, we can recover Brown's original experimentally determined frequency ratios free from any compounding of errors by just taking the ratios of the successive values of a in his empirical equation. For the successive frequency ratios for stage 2 to stage 1, 3 to 2, and 4 to 3, Brown's equation gives the respective values 2.3, 1.65, and 1.42. Curle's and Powell's equation (our equation 5) predicts the successive ratios 1.80, 1.44, and 1.36, which are rather different from Brown's values. Our equation 24 predicts the values 2.46, 1.82, and 1.45, which are close to Brown's values. It should be noted that the fact that Brown's equation predicts these frequency ratios correctly does not mean that the equation predicts frequencies correctly, that is, that the equation gives the correct dependence of frequency upon system parameters. The equation ignores the effect of slit width b upon edgetone frequency. All that would be required to make Brown's equation grossly in error in predicting frequencies while still predicting frequencies ratios correctly would be to use a greatly different value of b in Brown's experimental set-up while keeping v(0) and X the same. The frequencies observed experimentally would change since the transit time T(X) changes when b changes, so Brown's equation would predict the wrong frequencies, but it would still predict the frequency

ratios correctly (within experimental error, since equation 4 is an empirical equation based upon experimental data).

It will be noted from equation 11 that for sufficiently large values of slit width b (that is, when X is small compared to x0) the transit time T is effectively independent of b, and is a function of v(0) and X only. For this condition, the frequency of oscillation can be considered to be independent of the value of b and to be a function of v(0) and X only. For this condition the transit time T is very closely approximated by T = X/v (0). However, this is not generally true; when X is large compared to x0, then the value of b becomes important, and equation 11 must be used to determine the transit time.

There are two conditions to be satisfied if oscillations are to be produced in a feedback oscillator. One is a condition on gain or amplitude around the feedback loop, and the other is a condition on the phase change around the feedback loop. Since neglecting all amplitude factors in setting up and solving equation 16 makes it impossible to discuss the required gain in the oscillator, the amplitude factors in equations 17 through 20 have no absolute significance. But since we are doing a perturbation calculation, we have tacitly assumed that the maximum amplitude (i.e., y-displacement) reached by the wave while crossing the aperture to edge gap is some indefinite small value, large enough however to sustain the oscillations. The relative magnitudes of the y-displacement as wT increases, that is, as the wave traverses the aperture to edge gap, are very important though as the equations show that the amplitude of the wave increases as it traverses the gap, which is a well recognized feature of the jet-edge oscillator, a feature now predicted by our theory.

The phase condition ordinarily fixes the frequency of the oscillation. The phase condition is what we addressed when we assumed that the instant of fastest increase in the pressure of the acoustic field in a given halfspace coincided with the instant of greatest rate of loss of kinetic energy by the jet particles to that halfspace. This condition applied to our equation 19 gives equation 21 as a condition to be satisfied by the oscillator, which leads to equation 22, a condition on the fT product if oscillations are to occur. It is easily seen however that the condition fT = 0.500 not predicted by equation 21 also satisfies our assumption that the instant of the fastest increase of pressure in a halfspace coincides with the instant of the greatest rate of loss of energy by the jet particles to the halfspace. Adding this term to our fT sequence of equation 22 gives the final prediction of the fT sequence presented in equation 24.

For all but stage one the edge position is at a zero crossing of the jet wave y(wt, wT), defined by equations 19 and 20, for wt equal to zero or a multiple of 2*pi, but the basic assumption does not demand that all edge positions coincide with a zero crossing of this jet wave, this just happened to result in all stages but the first. There is no reason from the theory offered to conclude that this is a necessary condition for stage one. So there is no valid reason to doubt the prediction for the first stage just because one of the related circumstances is somewhat different. Every term in this final predicted fT sequence is a consequence of a single intuitively obvious assumption: the instant of fastest increase of pressure in a halfspace coincides with the instant of greatest rate of loss of kinetic energy by the jet particles to that halfspace.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## An Extended Comparison of Theoretical Predictions and Experimental Data

It is immediately apparent without numerical calculation, by approximating equation 11 for large and small values of b and x, that the predictions of edgetone behavior from equations 11 and 24 are in qualitative agreement with every aspect of the experimental behavior of edgetones pointed out above in Figure 1. For fixed b and X, the frequency is predicted to vary directly as v(0). For fixed b and v(0), the frequency is predicted to vary inversely as X for small X, and inversely as the three-halves power of X for large values of X. The fact that the jet is turbulent automatically introduces the three-halves power of X necessary to fit the

experimental data for wide gaps. For fixed v(0) and X, the frequency is predicted to increase as a function of b for small values of b, and to approach a limit as b becomes large, in agreement with CarriÃ¨re's data. The parameter b although universally neglected in previous theoretical attempts to explain edgetones is seen to be as important as the parameter X in determining the edgetone frequency. The constancy of the ratio of frequencies observed in different stages for the same set of parameters b, v(0), and X, is predicted. The empirical equations 1 through 4 for stage one are predicted by the first term of the sequence 24. In every case the predicted behavior is in qualitative agreement with the observed experimental facts shown in Figure 1. Additionally it will be shown later that the theory predicts the phase velocities of the jet wave near the edge that Brown measured.

It will now be shown that the theory is in almost exact quantitative agreement with the available experimental data. The best collections of experimental data are those of Carriere and Brown. The critical test of any theory is to predict the results of their experiments. The most extensive compilation of experimental data is that of Brown. The data of Brown's Table 1 will be exhibited in our Tables 2 and 3.

It has already been established by our Table 1 that the experimental values fT in each oscillation stage are in close agreement with the theoretical values in equation 24. For a further comparison with the experimental data of Brown's Table 1, the gap width X for each entry in Brown's table was calculated as a function of stage number, b, v(0), and f. The transit time T was first found from the sequence 24 using the stage number and observed frequency f. The gap width X was then predicted using equation 13. The comparison of the theoretical and experimental values of X is shown in our Table 2.

Table 2. Comparison of the theory with Brown's experimental data. The slit width b is 0.1 cm in all cases. With the stage number, b, v(0), and f as inputs to the theory, all gap widths X have been predicted. The numbers appended to X in the column headings indicate stage numbers. This table has the same arrangement of parameters as Brown's Table 1.

| | f (Hz) | X4 (cm) | X3 (cm) | X2 (cm) | X1 (cm) | v(0) (cm/s) |
|---|---|---|---|---|---|---|
| Experiment | 20 | -- | 5.68 | 3.92 | 2.02 | 137 |
| Theory | | 8.13 | 6.26 | 4.07 | 2.08 | |
| Experiment | 100 | 3.48 | 2.41 | 1.57 | 0.75 | 212 |
| Theory | | 3.51 | 2.67 | 1.69 | 0.82 | |
| Experiment | 150 | 3.33 | 2.50 | 1.64 | 0.79 | 309 |
| Theory | | 3.44 | 2.61 | 1.65 | 0.80 | |
| Experiment | 1200 | 1.74 | 1.28 | 0.79 | 0.34 | 984 |
| Theory | | 1.71 | 1.28 | 0.78 | 0.36 | |
| Experiment | 2400 | 1.58 | 1.15 | 0.75 | 0.33 | 1750 |
| Theory | | 1.56 | 1.16 | 0.71 | 0.32 | |

The agreement of the theoretically calculated values of the gap width X with Brown's experimental values is excellent. The calculation of Brown's Table 1 shown in our Table 2 does not critically depend upon the theory used to predict the theoretical fT sequence of equation 24. Our Table 1 justifies an empirical fT sequence fT = 0.50, 1.25, 2.25, 3.25, ..., which is so close to the theoretical sequence of equation 24 as to make little difference in calculating Brown's Table 1 using equation 13, with T from this empirical sequence

rather than from equation 24. It is therefore established empirically, as well as theoretically, that the edgetone oscillator is a transit time oscillator. To challenge this conclusion is to challenge not just the theory behind the predicted fT product sequence of equation 24, but to challenge either Brown's experimental data or Schlichting's equations for the slowing of a turbulent jet, since these two things alone suffice to give the fT product sequence and to predict Brown's Table 1. The validity of equation 24 is now established as an empirical fact, which is independent of any theory leading to that equation.

Every conclusion about the edgetone oscillator that will be stated in this paper is derivable just from Brown's data and Schlichting's equations, and is therefore an empirical fact. Objecting to the theory of this paper will not dispose of these conclusions since the conclusions can be established empirically independently of the theory of this paper. The principal equations 11, 13, and 24 of this paper, with the ancillary equations 9, 12, and 23, suffice to describe the edgetone oscillator, and do not depend upon anything in this paper for their validity.

The theory can be compared with Brown's data in other ways. Table 3 shows the comparison of the predicted values of the frequency with the experimental values. The transit time T was calculated for each of Brown's cases using equation 11, and the frequency was then found substituting this T into equation 24. The predicted frequencies are within a few percent of the experimental values.

Table 3. Comparison of the theory with Brown's experimental data. The slit width b is 0.1 cm in all cases. The first line in each group of three lists the experimental values of gap width X and initial jet velocity v(0) that give the experimentally observed frequency in the second line. The corresponding theoretically calculated frequencies are given in the third line.

| Stage Number | 4 | 3 | 2 | 1 | v(0) (cm/s) |
|---|---|---|---|---|---|
| Gap Width X, cm | -- | 5.68 | 3.92 | 2.02 | 137 |
| f experimental, Hz | -- | 20 | 20 | 20 | |
| f theoretical, Hz | -- | 23 | 21 | 21 | |
| | | | | | |
| Gap Width X, cm | 3.48 | 2.41 | 1.57 | 0.75 | 212 |
| f experimental, Hz | 100 | 100 | 100 | 100 | |
| f theoretical, Hz | 101 | 114 | 110 | 111 | |
| | | | | | |
| Gap Width X, cm | 3.33 | 2.50 | 1.64 | 0.79 | 309 |
| f experimental, Hz | 150 | 150 | 150 | 150 | |
| f theoretical, Hz | 157 | 159 | 151 | 152 | |
| | | | | | |
| Gap Width X, cm | 1.74 | 1.28 | 0.79 | 0.34 | 984 |
| f experimental, Hz | 1200 | 1200 | 1200 | 1200 | |
| f theoretical, Hz | 1180 | 1200 | 1210 | 1270 | |
| | | | | | |
| Gap Width X, cm | 1.58 | 1.15 | 0.75 | 0.33 | 1750 |
| f experimental, Hz | 2400 | 2400 | 2400 | 2400 | |

| | | | | |
|---|---|---|---|---|
| f theoretical, Hz | 2370 | 2440 | 2250 | 2340 |

............................................................. ........

Our equations do fit and predict Brown's experimental data. The agreement of the theoretical predictions with Brownâ€™s experimental data is excellent, within a few percent. The theory has no empirical or *ad hoc* elements introduced to force a fit to Brownâ€™s experimental data. Brown's data cover a very wide range of parameters, a factor of 120 for frequency f, 12.8 for initial jet velocity v(0), and 17.2 for gap distance X. The agreement of the theoretical predictions with Brown's experimental data is too close to allow a conclusion that the agreement is a fortuitous accident. It will be shown later that Brown's data on the phase velocity of the jet wave are also predicted.

Brown's data for the lower frequencies and wider gaps give values of the ratio (X/x0) appreciably greater than one, so therefore our equations 11 and 24 predict that the edgetone frequency in these cases should be exhibiting an inverse three-halves power dependence upon gap width X. That our equations predict Brown's data shows that this is the case. Brown's data therefore confirm what CarriÃ¨re and Jones both observed, that for some conditions the frequency varies inversely as the three-halves power of the gap width X.

The close agreement that has just been demonstrated of the predictions from this perturbational treatment of the edgetone problem with Brown's detailed experimental data proves the validity of our perturbational treatment. This was actually to be expected since the assumptions made in adopting this treatment are quite reasonable simplifications of the usual full hydrodynamic approach.

The theory will next be compared with CarriÃ¨re's data giving the frequency f as a function of slit width b with other parameters fixed. Unfortunately CarriÃ¨re gave the blowing pressure rather than the initial jet velocity so we are faced with the problem of converting blowing pressure to initial jet velocity. The conversion is not straightforward. In the general case where only the blowing pressure is known, Bernoulli's equation can not be relied upon to give an accurate prediction of the jet particle velocity. For Bernoulliâ€™s equation to apply it is necessary that the jet flow be steady, frictionless, and along a streamline, and that fluid density be a function of pressure only (Ref. 14). These conditions are not always, or even usually met. For a Borda tube aperture it has been shown theoretically that the initial jet velocity is 50 percent of the value predicted by Bernoulli's equation, for a sharp edged orifice the jet velocity is typically about 63 percent of the Bernoulli value, and for a Venturi orifice it can be close to 100 percent (Refs. 15). From the apparatus diagrams given by CarriÃ¨re, conversion factors of about 50 to 63 percent seem appropriate, and conversion factors in this approximate range give good agreement with our theoretical predictions while higher values would not. This conversion factor is an empirical factor we are forced to introduce in order to compare the theory with CarriÃ¨reâ€™s data.

Our Table 4 shows the comparison of theoretical predictions with CarriÃ¨re's data, using 66.4 percent as the conversion factor in determining the jet velocity from CarriÃ¨re's pressure values. The comparison was made in two ways. First, the theoretical values of T(X) were calculated using equation 11 and the resulting fT products were found for comparison with the predicted values of fT. The agreement with the value 0.500 predicted for stage 1 is almost exact, establishing stage 1 as the oscillation mode for Carriere's data. All of CarriÃ¨re's data appear to be for oscillations in stage 1. Second, using the calculated values of T(X) and assuming stage 1 oscillations, the expected frequencies were predicted using equation 24. The theory gives the right variation of frequency with slit width b for a 20 to 1 variation in b. Any error in the velocity conversion factor would appear as a constant scaling factor for the experimental fT products and the theoretically calculated frequencies.

Table 4. Comparison of theory with CarriÃ¨re's data giving edgetone frequency f as a function of slit width b. The gap X was fixed at 13.40 cm. The blowing pressure was 10 cm of water. The jet velocity is taken as 2690 cm/sec, or 66.4 percent of the value from

Bernoulli's equation.

| b (cm) | fT | | f (Hz) | | |
|---|---|---|---|---|---|
| Experiment | Exp. | Theory | Exp. | Theory | Error (percent) |
| 1.00 | 0.509 | 0.500 | 70.4 | 69.1 | -1.8 |
| 0.90 | 0.506 | 0.500 | 68.0 | 67.2 | -1.2 |
| 0.80 | 0.495 | 0.500 | 64.4 | 65.1 | 1.0 |
| 0.70 | 0.503 | 0.500 | 63.0 | 62.6 | -0.6 |
| 0.60 | 0.501 | 0.500 | 59.8 | 59.7 | -0.2 |
| 0.50 | 0.490 | 0.500 | 55.2 | 56.3 | 2.0 |
| 0.40 | 0.464 | 0.500 | 48.4 | 52.1 | 7.7 |
| 0.30 | 0.476 | 0.500 | 44.6 | 46.8 | 5.0 |
| 0.20 | 0.457 | 0.500 | 36.4 | 39.9 | 9.5 |
| 0.10 | 0.501 | 0.500 | 29.6 | 29.5 | -0.2 |
| 0.05 | 0.597 | 0.500 | 25.6 | 21.4 | -16.3 |

The velocity conversion factor was an empirical factor we had to introduce to apply the theory to CarriÃ¨re's data. However the theory then predicts CarriÃ¨re's results with very good accuracy as shown by the column giving the percentage error in the predictions.

CarriÃ¨re's experimental data show that the slit width b is fully as important as the parameter X in determining the edgetone frequency and that the slit width b should be an important parameter in any theory of the edgetone oscillator. CarriÃ¨re's data show that with v(0), X, and stage number fixed, a variation of slit width b by a factor of 20 to 1 caused a variation in edgetone frequency by a factor of 2.75 to 1. All previous theorists have failed to consider the dependence of edgetone frequency upon slit width b.

It is important to consider CarriÃ¨reâ€™s experiments in which the frequency varied inversely as the three-halves power of the aperture to edge distance. This is what the present theory predicts for wide gaps and a turbulent jet. Jones noticed that this variation occurred when the gap was wide and the jet was turbulent. Table 5 shows the comparison with CarriÃ¨re's data. T(X) was calculated using equation 11 for each value of X given by CarriÃ¨re. Then, assuming stage 1 oscillations, the expected frequencies were predicted using equation 24.

Table 5. Comparison of the theory with CarriÃ¨re's data for which the frequency varied inversely as the three-halves power of the gap width. The oscillations are identified as stage 1. The blowing pressure was 16 cm of water. The jet velocity is taken as 2455 cm/sec, or 48 percent of the value from Bernoulli's equation. The slit width is 0.25 cm. All numbers are CarriÃ¨re's except the theoretical values for f in the third column and the percentage values of the prediction errors.

| Experiment X (cm) | Experiment f (Hz) | Theory f (Hz) | Error (percent) | Experiment f*[X^(3/2)] |
|---|---|---|---|---|
| 16.7 | 28.2 | 29.2 | 3.5 | 1920 |
| 15.9 | 30.0 | 31.3 | 4.3 | 1900 |

| | | | | |
|---|---|---|---|---|
| 14.9 | 33.3 | 34.3 | 3.0 | 1910 |
| 13.9 | 35.2 | 37.8 | 7.4 | 1820 |
| 12.9 | 42.6 | 42.0 | -1.4 | 1970 |
| 11.9 | 47.6 | 47.0 | -1.2 | 1960 |
| 10.9 | 54.0 | 53.1 | -1.7 | 1940 |
| 9.9 | 62.0 | 60.6 | -2.3 | 1960 |
| 8.9 | 74.0 | 70.1 | -5.3 | 1940 |
| 8.1 | 80.0 | 79.6 | -0.5 | 1860 |

................................................................

The agreement shown is excellent as shown by the small percentage errors in the error column. The empirical factors used in converting blowing pressures to blowing velocities are within the range commonly found necessary (see Refs. 15) in applying Bernoulli's equation for this purpose.

The success of the present theory in predicting the results of critical edgetone experiments confirms this theory, and offers confirming evidence for the theory of jet motion presented in Schlichting's book. It suggests a new technique to determine the velocities of jet particles as a function of slit width b, initial jet velocity v(0), and distance X from the jet aperture.

These data of CarriÃ¨re's demonstrate conclusively that for some conditions the edgetone frequency varies inversely as the three-halves power of the gap width X. Brown's data and Jones' finding also confirm this. All previous theoretical efforts have ignored this strong three-halves power variation of frequency with gap width.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Comparison of Theoretical Predictions with More Recently Available Experimental Data

The author recently (summer, 2000) found on the Internet some very interesting experimental data on edgetones made available by Professor Yoshikazu Suematsu of Nagoya University, apparently abstracted from a published paper (Ref. 21), not yet seen by the present author. His data are very important since they greatly expand the range of v, f, X, and possibly aperture slit width b, covered by the easily available published data on edgetones. Although not stated in the limited material on the Internet, it appears that the fluid in his experiments was a liquid, but the theory of this paper applies to liquids as well as to gases. His curves published on the Internet give data for a single velocity v = 2.5 cm/sec showing frequencies varying from about 0.125 Hz to slightly over 0.375 Hz in stages 1, 2, and 3, for values of aperture to edge distances varying from about 3 cm to 20 cm. Unfortunately, the value of the slit width b corresponding to this data is not given. It will be assumed here that the data are for a single value of the slit width. Under the assumption that the theory of this paper is correct, a value for b, or equivalently for the parameter x0 of our theory, that fits all of Suematsu's data points, is easily deduced from a single data point. This fit to Suematsu's data justifies these assumptions.

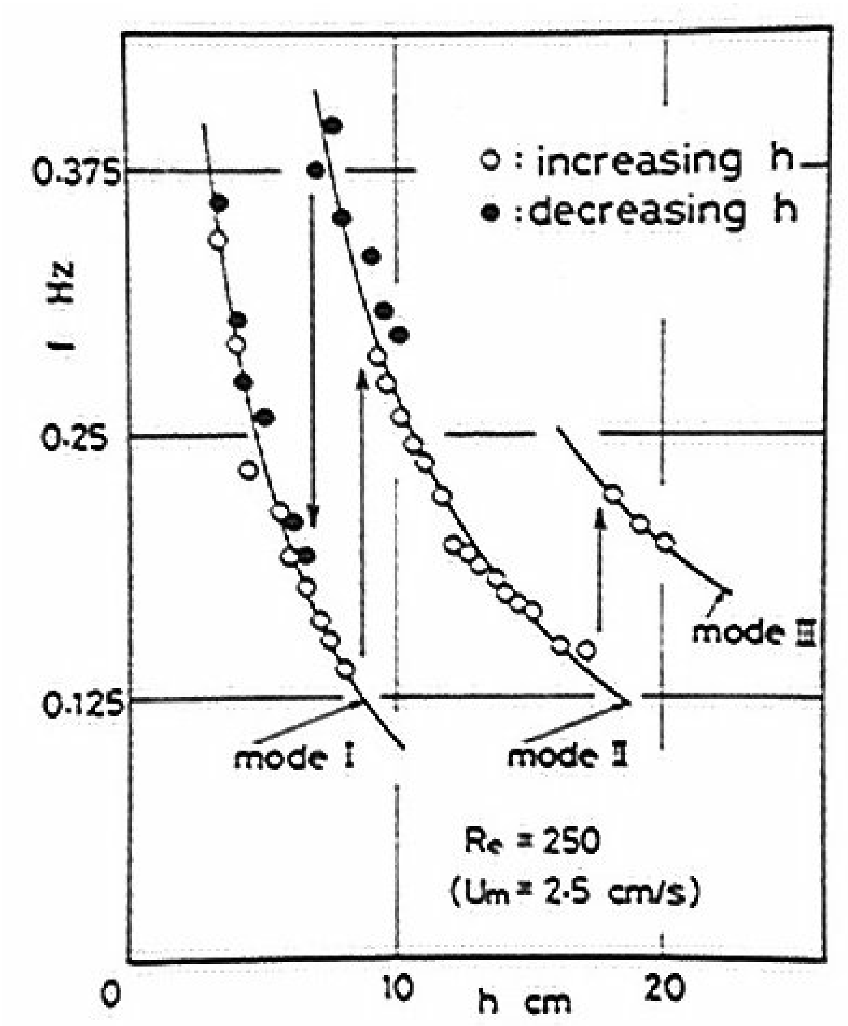
This material can be found at the Internet addresses:

http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/
http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/ET.html
http://www.suelab.nuem.nagoya-u.ac.jp/~suematsu/ET_fr.jpg

Our Figure 2 shown here is Suematsu's graph that presents his data that was just referred to. This graph as

shown here is found at the third of the addresses just given, and is found at the second address greatly reduced in size.

# Figure 2




Note that Suematsu uses the conventional symbol "h" for the aperture to edge distance where the present author uses the symbol "X". Table 6 shows the data for selected points as read by the author from Suematsu's curves in Figure 2. All entries in the table are Suematsu's experimental data except the entries labeled "f, our theory" and "f, Powell". Applying the theory of this paper to the case Stage II, v equal 2.5 cm/sec, X equal 18 cm, and f equal 0.125 Hz, we calculate the value 9.23 cm for the parameter x0 of our theory, or equivalently the value 1.61 cm for the slit width b. Using this value for the parameter x0, the theory of this paper predicts the frequencies designated "f, our theory" for the other cases shown in the table, which can be compared with Suematsu's experimental values. The frequencies designated "f, Powell" were calculated using Powell's empirical equation for the edgetone frequency (our equation 5).

............................................................

Table 6. The comparison of Suematsu's experimental data with theoretical predictions. All entries are experimental values read from Professor Suematsu's published graph, except for the calculated frequencies designated as "f, our theory" or "f, Powell". Those designated "f, our theory" were calculated by the

theory of this paper, assuming that the parameter x0 is 9.23 cm, or equivalently that the slit width b in Suematsu's experiments was 1.61 cm. Those designated "f, Powell" were calculated using Powell's empirical equation (our equation 5). [Note: the frequency entry in parentheses marked by an asterisk is the data point used to calculate x0; therefore the calculated value of frequency "f, our theory" for this case should be and is identical to the value "f, experiment".]

............................................................

| STAGE | 1 | 2 | 3 | |
|---|---|---|---|---|
| v(0) | 2.5 | 2.5 | 2.5 | cm/sec |
| | | | | |
| X | 9.0 | 18.0 | --- | cm |
| f, experiment | 0.125 | (0.125)* | --- | Hz |
| f, our theory | 0.114 | 0.125 | --- | Hz |
| f, Powell | 0.174 | 0.156 | --- | Hz |
| | | | | |
| X | 5.0 | 11.0 | 16.0 | cm |
| f, experiment | 0.250 | 0.250 | 0.250 | Hz |
| f, our theory | 0.222 | 0.223 | 0.257 | Hz |
| f, Powell | 0.313 | 0.256 | 0.254 | Hz |
| | | | | |
| X | 3.0 | 8.0 | --- | cm |
| f, experiment | 0.375 | 0.375 | --- | Hz |
| f, our theory | 0.387 | 0.322 | --- | Hz |
| f, Powell | 0.521 | 0.352 | --- | Hz |
| | | | | |
| X | 10.0 | 10.0 | --- | cm |
| f, experiment | 0.106 | 0.266 | --- | Hz |
| f, our theory | 0.101 | 0.249 | --- | Hz |
| f, Powell | 0.156 | 0.281 | --- | Hz |
| | | | | |
| X | --- | 20.0 | 20.0 | cm |
| f, experiment | --- | 0.106 | 0.194 | Hz |
| f, our theory | --- | 0.108 | 0.196 | Hz |
| f, Powell | --- | 0.141 | 0.203 | Hz |

..............................................................

Powell's empirical equation fails very badly in all of its predictions of Suematsu's frequencies for stage 1, and in its prediction of Suematsu's frequency for stage 2 for the gap distance X equal to 20 cm.

It is evident from Table 6 that the theory of this paper predicts with very good accuracy the frequencies seen by Suematsu in his experiments. This comparison of the theory with the experimental data required that the parameter x0 of the theory be calculated using the data of one of Suematsu's data points in order to predict the frequencies seen in all the other data points. The prediction of these other frequencies is very good. However, even without a value for the parameter x0 (or of the slit width b), it is possible to compare certain predictions of the theory with Suematsu's data; this will be demonstrated.

We will recall that for the same set of parameters v(0), b, and X applied to any two stages of oscillation, our theory predicts that the ratio of the two frequencies seen in the two different stages is a function only of the

two stage numbers and is independent of the particular values of v(0), b, and X. All of Suematsu's data is for a single value of jet velocity v(0) and a single value of aperture slit width b. Therefore our theory, as presented here without any adjustments, should predict the ratio of the two frequencies that Suematsu saw in two different stages for the same value of aperture to edge gap distance X. For the aperture to edge gap distance X equal to 10.0 cm in both stage 2 and stage 1, the ratio of the frequency seen in stage 2 to the frequency seen in stage 1, as calculated from the experimental data in Table 6, is 2.51; for the gap distance X equal to 20 cm in both stage 3 and stage 2, the ratio of the frequency in stage 3 to the frequency in stage 2 is 1.83. The corresponding ratios predicted by the theory of this paper are 1.230/0.500 = 2.46 and 2.239/1.230 = 1.82. The theoretical value 2.46 is to be compared to the experimental value 2.51, and the theoretical value 1.82 is to be compared to the experimental value 1.83. The agreement of the predictions with Suematsu's data is essentially perfect. In contrast, the ratios predicted from Curle's and Powell's frequency equation for the edgetone oscillator (our equation 5) are 1.80 and 1.44, very different indeed from the ratios seen in Suematsu's experiments. As pointed out in detail in this paper, the equation of Curle and Powell is based upon a serious misinterpretation of Brown's experimental data by Brown, Curle, and Powell, and is therefore unsupported by experimental data.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Jet Wave Phase Velocity

The present theory predicts that the phase velocity of the jet wave is not a fixed quantity across the gap as tacitly assumed by Brown and accepted by later theorists, but is a function of position in the gap. Equation 20 demonstrates this most conveniently. For values of wT small enough that sin(wT) can be approximated by wT, equation 20 becomes

```
y(wt, wT) = wT [cos(wt - wT) - cos(wt - wT/2)]                (25)
```

which in turn with the same approximation for sin(wT) reduces to

```
y(wt, wT) = [(wT)^2] * sin(wt - 3wT/4)                        (26)
```

The last equation indicates that the phase velocity at any point very near the aperture is four-thirds times the jet particle velocity at that point. This is perhaps more obvious if T in the sine term of equation 26 is replaced by its equivalent as a function of X given by equation 11.

For values of wT greater than about 2, or for values of fT greater than about 0.318, the first term of equation 20 dominates its behavior and the second term can be neglected. Equation 11 is then approximated by

```
y(wt, wT) = (wT)cos(wt - wT)                                  (27)
```

This first term taken alone indicates a phase velocity at a point equal to the jet particle velocity at that point. Therefore for values of fT greater than about 0.318, the phase velocity at a point is almost equal to the jet particle velocity at that point. The phase velocity at the position x(T) in the gap should then be closely approximated by v(x) where v(x) is given by equation 8. Both jet and phase velocities are functions of position in the gap. The phase velocity of the jet wave is never less than the velocity of the jet particles. Brown's and later theorists' conclusion that the phase velocity of the jet wave is only a fraction of the jet particle velocity is wrong.

Since the jet particle velocity v(x) becomes smaller as the jet particles approach the edge, the phase velocity of the jet wave also becomes smaller as the edge is approached. This means that the wavelength seen in the

jet wave also becomes smaller as the edge is approached. Therefore Brown's conclusion that the number of wavelengths of the jet wave in the aperture to edge gap was equal to the gap width divided by the wavelength measured just before the edge is incorrect and leads to a serious overestimate of the number of wavelengths in the oscillation of the jet in the gap. This then means that Curle's and Powell's equation (our equation 5) for the frequency of the edgetone oscillator has no support in Brown's data, is therefore an unsupported conjecture, and is wrong.

These conclusions of the present theory about the phase velocity of the jet wave are very different from those of previous theorists, who neglected the slowing of the jet particles and tacitly took the jet velocity anywhere in the gap as always equal to the jet velocity at the aperture. The phase velocity of the jet wave was believed to be about half that value anywhere in the gap, as exemplified by the common interpretation of equations 3 and 4, Brown's interpretation of his experimental data, and Curle's and Powell's interpretation of equation 5.

It will not be demonstrated here but the present theory can predict without approximation the wavelengths measured by Brown, and therefore the phase velocities which Brown found. Equation 21 predicts the sequence of jet travel times T that correspond to all the zero crossings of the jet wave in the gap for the values of wt equal to zero or to some multiple of 2*pi. For the purpose of calculating wavelengths, the value T equal to zero should be taken as the first member of this sequence of values of T. This sequence of values of T substituted into equation 13 predicts the sequence of values of position x in the gap that correspond to successive zero crossings of the jet wave, with the value x equal to zero being the first value of x in the sequence. Presumably, as seen by Brown, the distance between successive zero crossings is one-half wavelength, and the distance between alternate zero crossings is one complete wavelength. Starting from the value x equal to zero, every second value of x in this sequence x of zero crossing values is a value of X in the sequence of edge positions X that give rise to edgetone oscillations in stages two and higher. The value of X for stage one is not a member of this sequence. We will arbitrarily, for convenience, include the value X equal to zero in this sequence of values X giving rise to edgetone oscillations, although of course no oscillations occur for this value of gap width X. We will designate the sequence of values of X that results by

```
X(I) = X(0), X(2), X(3), X(4), ...                                (28)
```

where, except for X(0), the value X = X(I) is a value of aperture to edge distance that gives rise to edgetone oscillations in stage I. X(1) is not a member of the sequence X(I) defined in this manner.

Define Lambda(I) to be the wavelength seen by Brown for stage I. Then for stage numbers I equal two and higher the wavelengths Lambda(I) seen by Brown are given by

```
Lambda(2) = X(2) - X(0)
(29)
```

```
Lambda(I) = X(I) - X(I - 1);  I = 3, 4, 5, ...                    (30)
```

These wavelengths Lambda(I) are just the successive differences between the values of X in the sequence X (I) of equation 28. These are the only wavelengths that Brown could define. He could not experimentally define a wavelength in stage one. The wavelengths Brown saw are effectively just the increments in aperture to edge distance X between successive oscillation stages, although Brown did not recognize this. This procedure does not give a wavelength prediction for stage one, but stage one oscillations are a special case

arising from the value fT = 0.500 not predicted by equation 20, and a zero crossing of the jet wave does not occur for the value of x (or X) which corresponds to the value of T for stage one. The derivation of the product fT for stage one indicates that we should not expect to see a complete wavelength in stage one, and indeed the experimental fact is that Brown could not define a wavelength in stage one. Therefore our theoretical procedure predicts this aspect also of Brown's data.

It is much simpler to do the equivalent of the process just outlined by taking the phase velocity of the jet wave at a point as equal for practical purposes to the jet particle velocity at that point, so that wavelengths at a point are closely approximated by dividing the jet particle velocity at that point by the edgetone frequency, except for points very near the jet aperture.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## The Comparison of Predicted and Experimental Values of Phase Velocity

Brown's procedure gives the phase velocity not at a point but averaged over one wavelength just before the edge. Although Brown did not state it this way, obviously what Brown determined was the ratio of the phase velocity of the jet wave averaged over one wavelength *immediately adjacent to the edge* to the jet particle velocity *at the aperture*, and stated this way Brown's experimentally determined ratios are valid numbers, that is, they are correct for Brown's particular experiment, but there is no *a priori* reason to assume that these same ratio values for phase velocity at the edge to particle velocity at the aperture would be valid for experimental parameters different from Brown's. A more obvious comparison would have been to compare the phase velocity at the edge to the jet particle velocity also at the edge, but this is not the comparison which Brown made. Brown implicitly assumed that although the two velocities were measured at different points in his experimental setup, the ratio so determined had universal validity. There is no obvious justification for such an assumption, and in fact the assumption is seriously wrong. For the higher stages, for which the wavelength is much shorter since there are more wavelengths in the gap and the jet has slowed appreciably, Brown's average is taken over a short distance just adjacent to the edge. The present theory predicts that, except very near the jet aperture, the phase velocity of the jet wave at a point is very nearly equal to the jet particle velocity at that point. Therefore, Brown's experimentally determined ratios, for the higher stages, of the phase velocity near the edge to the jet particle velocity at the aperture, should differ only slightly if at all from the theoretically calculated ratio v(X)/v(0) at the edge. And this is indeed the case. For every entry in Brown's Tables 2 and 3 our equation 8 gives v(X)/v(0) = 0.38 at the edge. The jet particles in crossing from the jet aperture to the edge have lost over sixty percent of their initial velocity. Of the nineteen valid entries in Brown's Table 2 for the ratio of phase velocity immediately before the edge to jet particle velocity at the aperture, eighteen are in the range 0.36 to 0.43, in close agreement with our prediction. One entry gives 0.47 for the ratio. (There are twenty entries in the table but one has to be discarded because it is for stage 1 and is not experimental data but is a guess by Brown, this entry should not have been included in Brown's table.) For the highest stage in Brown's Table 3, the ratio values are 0.40, 0.41, and 0.38, again in agreement with our prediction of 0.38 for what Brown would see for the higher stages. Schlichting's equation for the slowing of a turbulent jet (our equation 8) gives an accurate prediction of the phase velocities Brown observed. If Brown or others had calculated the slowing of the jet particles, they would have recognized these phase velocities as being for practical purposes just the velocities to which the jet particles had slowed. Brown's experimental data are entirely consistent with the conclusions of the present paper about the phase velocity, in fact, the present theory predicts the phase velocities which Brown found. We are forced to the conclusion that while Brown's experimental data are excellent, serious errors have been made in the interpretation of these data. Brown tacitly assumed that the jet particle velocity at any point in the aperture to edge gap was equal to the jet particle velocity just out of the aperture, and that the phase velocity of the jet wave at any point in the gap was a small constant fraction of that constant jet particle velocity. These errors of interpretation contributed to previous failures to develop a satisfactory theory.

It will be noted that the conclusion of this paper that at points not very near the aperture the phase velocity of the jet wave at those points is essentially equal to the jet velocity at those points is now an empirically established fact, not dependent upon the theory of this paper. If Brown's basic data are correct and Schlichting's equations for the slowing of the jet are correct, then this conclusion is correct. The conclusion follows from Brown's data and Schlichting's equations, neither of which depends upon anything in this paper.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Critical Review of Past Theoretical Efforts

No past theoretical effort to explain edgetones could even attempt to predict all the basic phenomenology of edgetones outlined above in Figure 1, and certainly not the detailed experimental data of CarriÃ¨re and Brown, so the only detailed comparison with past work that is possible is to discuss their inferences about the phase velocity of the jet wave in the gap. It will be shown that Brown's data on the phase velocity of the jet wave have been misinterpreted by Brown and others, and that Brown's data are predicted by the present theory.

Karamcheti, Bauer, Shields, Stegen, and Woolley in 1969 (Ref. 16) discussed briefly the availability of experimental evidence on the phase velocity of the jet wave in the aperture to edge gap. They state *"the only experimental information on the phase criterion is that indicated by Brown's measurements of the wavelength from smoke pictures of the oscillating jet."* Brown injected smoke into the jet and photographed the jet disturbance in the gap. There was not enough detail in the photographs to allow the number of wavelengths in the jet disturbance in the gap to be counted. He was able to define and measure a wavelength in the jet disturbance in these photographs only near the edge and only for the higher stages. He states that photographs of stage one were such that no wavelength could be defined. For the other stages he could only make measurements near the edge since insufficient details of structure were discernible in earlier parts of the jet path. Great accuracy should not be expected in these measurements. The measured wavelength multiplied by the frequency gave a phase velocity which was about one-fourth to one-half of the velocity Brown attributed to the jet, which was the velocity of the jet at the aperture. Brown did not tabulate the phase velocities themselves but instead gave the ratios of the phase velocities he determined to the initial jet velocity at the aperture. The ratios Brown determined are valid numbers, but Brown misinterpreted their significance. Brown's values for the wavelength are valid only immediately before the edge. There are the implicit assumptions in Brown's treatment of his data that the jet does not slow down and that the wavelength in the jet disturbance in the gap is the same at all positions in the gap, and therefore that both the jet particle velocity and the jet wave phase velocity are constant independent of position in the aperture to edge gap. Brown concluded then that the phase velocity (assumed constant) was a small fraction of the jet particle velocity (also assumed constant). There is no basis in theory or experiment for Brown's assumptions. It is known that these jets do slow down rapidly. However later theorists neglected the known slowing of the jet and uncritically accepted Brown's interpretation of his data. Karamcheti and Bauer (Ref. 17; also see Ref. 16, page 295) have noted that assuming a single disturbance wavelength and propagation velocity throughout the edgetone jet is not correct, but their remark seems to have passed unnoticed by most theorists.

There are other major problems with Brown's presentation and interpretation of his experimental data. Brown could not experimentally define a wavelength for stage one oscillations. However he assumed that for stage one the wavelength was just the aperture to edge distance, and he calculated values for phase velocity in stage one on the basis of this assumption. In his text he stated that he had done this. Unfortunately, and inexplicably, he included these assumed and calculated values for stage one in his table

of real experimentally determined values for the other stages, without an explicit warning in the table that for stage one the values given had not been experimentally determined but were in effect guesses. This entry should not have been included in Brown's table of experimental data. This is the entry in Brown's Table 2 that was rejected from consideration in the paragraph above. Brown had no experimental value of wavelength or phase velocity for stage one. Strangely, the existence of this invalid entry in Brown's table of data has not been previously pointed out. It is an unfortunate fact that past theorists, subsequent to Brown, using these data overlooked Brown's statement of what he had done and treated these particular values for stage one as valid experimental data. Curle and Powell both made this error, and each of the many subsequent investigators of edgetones uncritically accepting Brown's presentation and interpretation of his data, or following Powell's lead in this matter has unwittingly contributed to the perpetuation of this persistent error that the phase velocity of the jet wave is a small fraction of the jet particle velocity. This inattention to what Brown wrote and the resulting confusion about Brown's data have undoubtedly contributed to the failures to develop an adequate theory.

The lack of detail except near the edge in Brown's photographs made it impossible to count the number of wavelengths in the oscillation of the jet in the gap. The number assigned by Brown for stage one and entered into his table of what was otherwise numbers derived from experimental data, one complete wavelength, was a pure guess as we saw in the paragraph immediately preceding. This entry must be disregarded. Except for stage one, the numbers assigned were obtained by dividing the gap width by the wavelengths measured just before the edge, which was the only place with sufficient detail to allow the wavelengths to be defined and the measurements made. This procedure tacitly assumed that the wavelength was constant and independent of position in the gap. The measured wavelength values of course are valid but must be properly interpreted. There is neither experimental nor theoretical justification for Brown's assumption that the wavelength measured just before the edge was the wavelength at all positions in the gap. This gave much too large a result for the number of wavelengths in the gap because the wavelength in the jet gets smaller as the edge is approached since the jet is slowing down. Every entry Brown gives for the number of wavelengths in the gap must be rejected. Theorists making uncritical use of Brown's data who accepted his interpretation of that data were led to incorrect conclusions. For every stage Brown assigned one complete wavelength too many for the integer part of the number of wavelengths in the jet disturbance in the gap and the fractional part of this assignment above an integer has no validity either. Curle and Powell accepted this assignment as the basis for postulating the empirical equation they proposed (our equation 5). It is evident that the values Brown tabulates for the number of wavelengths in the oscillation of the jet in the gap have no validity whatever, and offer no confirmation or support to the equation that Curle and Powell proposed for the edgetone frequency. That equation is completely devoid of experimental support by Brown's data and is therefore an unsupported conjecture. All subsequent theoretical papers (and there are many) following Powell's lead and adopting this equation are based on the false premise that this equation is supported by Brown's experimental data. Therefore these papers must be rejected or at least critically reexamined. Brown's and later theorists' interpretation and treatment of Brown's excellent experimental data were unchallenged until the present paper.

Powell has probably been the most influential writer on edgetones since Brown, and certainly the most prolific. He has been the major champion since Brown of the viewpoint that the phase velocity of the jet wave is everywhere a small fraction of the jet particle velocity. Powell has continued to publish in various journals numerous articles and letters on edgetones (too many to reference all here, but see for example Ref. 18) offering elaborations of the viewpoint and equation he first presented in 1953. Although Powell's equation (equation 5 above) is obviously inadequate and cannot predict or explain the detailed data of Brown, and certainly not the data of CarriÃ¨re, Powell has continued to staunchly support this equation and his original approach to this problem. The influence of Powell upon other theorists has been very great, which in most respects is unfortunate. The continued acceptance of Brown's assumptions and interpretation of his data by Powell vitiates most of Powell's work on edgetones. Powell's approach to the problem of edgetones and his equation have been adopted (as recently as 1998, see Ref. 20) as a starting point by many later theorists attempting to explain edgetones. It will be apparent that the present paper completely invalidates Powell's and his followers' approach to the problem of edgetones. (Note added in 1999: Powell's

and his followers' efforts to explain edgetones, continuing to depend upon Brown's assumptions and interpretation of his data, now cover a period of almost fifty years, and have yet to produce either a theory or an empirical procedure that can predict the experimental data of CarriÃ¨re and Brown.)

It truly seems that the major reason for the long failure to have a successful theory of edgetones has been the uncritical acceptance by substantially all theorists after Brown of the faulty interpretation Brown offered of his data. Powell and following theorists just did not consider that Brown might have been seriously wrong in his treatment of his data. However even a cursory reading of Brown's paper shows that the treatment and interpretation that Brown offers of his experimental data are dependent upon two unstated assumptions, that the jet particles do not slow down in crossing the aperture to edge gap, and that the wavelength of the jet wave measured immediately before the edge is the wavelength at all positions in the aperture to edge gap. The existence of these unstated and incorrect assumptions should have been recognized long ago. Brown certainly gives enough information about how he interpreted his experimental data to allow these unwarranted assumptions to be easily identified. But since Brown's paper, nearly every paper on edgetones or the organ flue pipe has tacitly assumed as a starting point that the jet particle velocity does not vary across the gap, and that the phase velocity of the jet wave in the gap is only a fraction of the jet particle velocity in the gap. These papers are immediately seriously in error.

To summarize, the phase velocity of the jet wave in the edgetone oscillator and in musical instruments such as the flute and organ flue pipe is not a small fraction of the jet particle velocity, as stated to be the fact in most texts and papers dealing with this oscillator and these musical instruments. To slightly adapt here a short but very appropriate quote from T. Needham written about a different subject but for a similar situation (T. Needham, Visual Complex Analysis, Oxford University Press, 1997, p. 386): *"We have made a fallacy of an assertion that is to be found in most texts. Perhaps the mere frequency with which this myth has been reiterated goes some way to explaining how it has acquired the status of fact."*

Theorists attempting to explain edgetones have also neglected large parts of the known phenomenology of edgetones. They have not given attention to the finding of CarriÃ¨re, confirmed by Jones and also deducible from Brown's data, that for some conditions the edgetone frequency varies inversely as the three-halves power of the gap width X, or to the finding of CarriÃ¨re, confirmed by Brown, that the edgetone frequency depends upon the aperture slit width b. These dependencies are too strong to be ignored in an adequate theory of the edgetone oscillator. No prior theoretical paper known to the author gives serious consideration to CarriÃ¨re's work, acknowledges that a three-halves power dependence of frequency on gap width X exists which must be explained, or makes the slit width b an important parameter of the theory proposed. None takes account of the possible effects of jet slowing upon edgetone production.

In many of the prior theoretical papers on edgetones vortices shed from the jet wave at the aperture, traveling across the aperture to edge gap, and interacting with the edge have been assumed to be the causative agent giving rise to the edgetones. None of these papers however has presented a quantitative explanation of just how this postulated interaction gives rise to edgetones or presented a theory that can predict the experimental data on edgetones, so it is not established as a fact that vortices are the causative agent of edgetone production or even a significant factor in edgetone production. That has remained a theoretical speculation. No mention of vortices as contributing to edgetone production has been found necessary in the present paper. The causative agent for edgetone production has been identified in this paper as the interaction between the moving jet particles and the oscillating acoustic pressure field of the oscillations. That is a very satisfactory outcome as it reduces the explanation of edgetones to the same explanation that accounts for the operation of comparable electronic oscillators. Ultimately the explanation of the operation of any electronic oscillator is the interaction of an electric current with an oscillating electric field. An analogous explanation seems to hold true for any acoustic oscillator. The current has to interact with the field in order to add energy to the field, which is a necessary condition if oscillations are to be produced. The production of vortices may indeed be a phenomenon that accompanies the production of edgetones, but the vortices do not appear to be the cause of the edgetones, they are instead an accompaniment or consequence of edgetone production. In support of this we note that Schlichting's

equations for a turbulent jet which are certainly valid when no oscillations are occurring give no indication that vortices are present or being produced. All that is necessary to convert Schlichting's equations for the turbulent jet to our equations for the jet wave of the edgetone oscillator is to introduce the oscillating acoustic pressure field *sin(wt)*.

The theory of the present paper has aroused considerable opposition among those imbued with the prevailing opinion expressed in virtually all past work that the phase velocity of the jet wave is only a fraction of the jet particle velocity. It is evident that the prevailing opinion is based on the two unstated assumptions that (1) the jet particle velocity is constant everywhere in the gap and always equal to its value at the aperture, and (2) the phase velocity of the jet wave is constant everywhere in the gap and always equal to its value measured just adjacent to the edge. There is no basis for the tacit assumptions that these velocities are constant across the gap, and therefore that the ratio of phase velocity to jet particle velocity is a constant independent of position in the gap which is equal to the ratio of phase velocity just adjacent to the edge to the jet particle velocity just out of the aperture. In the eighty years since Krueger's paper no one adopting his view of the phase velocity of the jet wave has been able to predict the observed data on edgetones. The present theory with no *ad hoc* elements does predict Brown's and CarriÃ¨re's data on edgetones with exceedingly good accuracy, including Brown's data on the phase velocity of the jet wave.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## A Recommendation for Modern Experiments

It is a strange fact that present day theorists discussing edgetones still must base their discussions mainly upon the experimental results of Brown published (as of the year 1999) more than sixty years ago, in what we might consider to be "the dark ages" of instrumentation development and certainly still "the dark ages" of our understanding of edgetones. Also there is an important lack in Brown's data. Although it was known from CarriÃ¨re's prior work on edgetones that the aperture slit width b was just as important as the initial jet velocity v(0), gap width X, and the stage number in determining the edgetone frequency f, Brown did not vary the slit width b in his reported data. The result has been that modern theorists hardly give attention to the experimental parameter b as being important in determining the edgetone frequency. I know of no prior paper on edgetones other than CarriÃ¨re's that recognizes the great and equal importance of the aperture slit width in determining the edgetone frequency.

Modern experiments on edgetones, guided by our present understanding of edgetone phenomenology such as is exhibited in this paper, and using modern technologies would be very useful. A repetition of Brown's experiments, with the slit width b being one of the parameters varied in addition to v(0), X, and the stage number should be carried out. The range of variation of the parameters could also be increased with no great difficulty using modern technology. Although actually Brown's data have been shown in this paper to be sufficient to verify the present theory, new experiments designed specifically to test the theory would put the matter beyond cavil.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## Summary

This paper establishes as an empirical fact independent of any theory offered in this paper that the edgetone oscillator is an acoustic transit-time oscillator governed by the equation fT = k where T is the transit time of

a jet particle from the jet aperture to the edge, and k is a constant that depends upon the oscillation mode. The values of k are established by presently available experimental data. The paper also offers a theory that predicts the value of k seen for each oscillation mode.

The theory presented here predicts with extremely good accuracy the available experimental data on edgetones without the introduction of any empirical factors intended to force a fit to that data. It has the virtue that it shows that the edgetone oscillator is just another simple oscillator of well known type, explainable by the same principles of oscillator operation that govern a multitude of familiar electronic oscillators. The only critical feature in the presentation is the adoption of a perturbational approach to the problem, and that approach has been fully validated by the result.

This theory was developed in the years 1971 through 1973. A brief account of the work was presented at the Los Angeles meeting of the Acoustical Society of America, 30 October-2 November, 1973; and an abstract (Ref. 19) of that presentation was published in 1974. The key equations of this paper, equations 9, 11, 12, 13, and 24, are presented in the abstract. The abstract's statement that the phase velocity of the jet wave in stage one is twice the jet particle velocity there is incorrect. The error arose from the hasty assumption that only the second term of equation 20 was important for very small values of wT. (The phase velocity of the jet wave at a point very near the aperture is four-thirds times the velocity of the jet particles at that point.)

A first version of this paper was written in early 1972. The present version was written in 1974. Revisions were made in 1999 and 2000. The original paper stated only casually that the oscillation would be considered as a perturbation to the motion of the jet. A major purpose of the recent revisions was to emphasize that the theory being presented was a perturbation theory, and to show that this approach was fully justified. Text to that purpose has been added. No equations and no conclusions were changed. One new equation, equation 7, was introduced. The literature since early 1974 has not been reviewed. A detailed account of the theory has not been previously published.

The most important conclusion of the present paper is that the edgetone oscillator is an acoustic transit time oscillator, not different in principle from many electronic oscillators. Perhaps equally or even more important is that erroneous assumptions in Brown's presentation and interpretation of his excellent experimental data are identified. These assumptions tacitly adopted by Brown and unquestioningly accepted by later theorists relying upon Brown's paper have had the most serious consequences upon efforts to develop an understanding of the edgetone oscillator.

(Note added in 1999: One recent theorist on edgetones, Young-Pil Kwon (Ref. 20), in two interesting papers has independently noted in referring to Brown's paper that *"the jet velocity decreases with distance along the jet axis"* and that *"the wavelength measured near the edge tip may be shorter than the average wavelength along the stand-off distance"* in consonance with this paper. Although not greatly emphasized by him, this amounts to a denial of Brown's assumptions about the jet and phase velocities. He did adopt Powell's equation (our equation 5) as a starting point for his discussion of the edgetone problem, but recognized that as a guide to edgetone oscillator behavior this equation would require major modifications.)

Although usually unstated, implicit in most previous theoretical efforts to explain edgetones is the tacit assumption that the jet velocity remains constant all across the gap from aperture to edge. It is usually also explicitly assumed that the jet wave is characterized by a constant phase velocity which is a small fraction of that assumed constant jet velocity. The first assumption is contradicted by the known facts about jet behavior, and the second assumption is without support in experiment or theory. No theory based on these assumptions has had success in predicting the data from basic edgetone experiments, despite a plethora of theoretical efforts extending over most of a century. None gives even a qualitative prediction of all the basic phenomenology of edgetones outlined above.

None of these previous theoretical efforts has given attention to or attempted to explain either qualitatively or quantitatively the three-halves power variation of frequency with gap width seen in some experiments, or

the strong variation of frequency with slit width which is known to exist and for which quantitative data is available. The slit width is not even a parameter of these theories. Despite claims of validity that have been made for these theories, none has been successfully applied to predict the data of CarriÃ¨re and Brown. These data exist, their validity has not been challenged, and a successful theory must explain them. The author regards the predictions of these data as the basic tests of any edgetone theory. The theory of the present paper is the only theory of edgetones yet developed that predicts and explains the results of these basic experiments.

The theoretical situation is further clouded by the fact that a clear distinction is not always made by theorists between the theory of edgetone oscillations and the theory of flute or organ pipe oscillations. An adequate theory of the organ flue pipe will have little application to the edgetone oscillator. The impact of the Q-factor of the flute's or organ pipe's resonant air column upon the oscillation frequency of the musical instrument is so strong that a clear distinction between the two theories should be made. Any theory of edgetones has direct bearing upon the theory of flutes or organ pipes since the edgetone appears to be the exciting agent of the flute or organ flue pipe. However the Q-factor of the musical instrument so modifies the behavior of the edgetone oscillator as to make separate discussions of the two oscillators at least desirable if not strictly necessary. (See the discussion of Lord Rayleigh's data on the variation of frequency of an organ flue pipe with changes of blowing pressure, which can be found at http://www.nmol.com/users/wblocker/index.htm where the present paper is also found. Lord Rayleigh's data shows that changes of blowing pressure, and consequently of blowing velocity, that would more than triple the frequency of an edgetone oscillator make a change of only a few percent in the frequency of an organ flue pipe oscillator. This stabilization of the frequency is attributable to the Q-factor of the organ flue pipe. Even modest values of the Q-factor have very great effects in stabilizing the frequency sounded by the pipe. Any parameter having an influence this great upon the frequency sounded by the flue pipe should receive great attention in any theory of the flue pipe. Most theories of the organ flue pipe oscillator fail to mention the pipe's Q-factor and the few that do give scant attention to its importance.)

The assumption that the edgetone oscillator is a transit time oscillator leads immediately to a theory that predicts all the basic phenomenology of edgetones and gives numerical predictions in almost exact agreement with all critical experimental data. This is the first theory that can predict in detail the experimental results of Brown and CarriÃ¨re. But the theory is not essential. Every important conclusion of this paper follows from Brown's experimental data and Schlichting's equations for the slowing of a jet. These alone suffice to establish empirically the fT product sequence, thus to establish empirically that the edgetone oscillator is a transit time oscillator, and to establish empirically that, except possibly very near the aperture, the phase velocity of the jet wave at a point is equal for practical purposes to the jet particle velocity at that point. These things are now empirically established facts, and are independent of the theory that led to them.

The major factor leading to this theory was the conviction that the edgetone oscillator must be an ordinary feedback oscillator which could be explained in the same fashion that all other feedback oscillators are explained. With this conviction, the feedback loop is easily identified and analyzed. In fact Jones had already identified the feedback loop and the feedback mechanism. The edgetone oscillator is an acoustic feedback oscillator, more specifically a transit time oscillator. The calculation of the transit time must take account of the slowing of the jet particles and take account that the jet is turbulent. The same principles that explain the operation of ordinary electronic feedback oscillators suffice to explain the operation of this oscillator.

Return to the Table of Contents

Return to The Home Page of Wade Blocker.

## References

(1) C. Sondhaus, Ann. Phys. 91, 128 & 214 (1854)
(2) J.M.A. Lenihan and E. G. Richardson, Phil. Mag. 29, 400 (1940)
(3) W. Koenig, Phys. Z. 13, 1053 (1912)
(4) E. Schmidtke, Ann. Phys. 60, 715 (1919)
(5) F. Krueger, Ann. Phys. 62, 673 (1920)
(6) M.Z. CarriÃ¨re, J. de Phys. et Rad. 6, 52-64, (1925)
(7) G.B. Brown, Proc. Phys. Soc. 49, 493 & 508 (1937)
(8) A. T. Jones, J. Acoust. Soc. Am. 14, 127(A) (1942/1943)
(9) N. Curle, Proc. Roy. Soc. A216, 412 (1953)
(10) A. Powell, Acoustica 3, 233 (1953)
(11) Prandtl and Tietjens, Fundamentals of Hydro- and Aeromechanics, Chapter XV, pp. 251 et seq., Dover Publications, Inc., New York, 1957
(12) H. Schlichting, Boundary Layer Theory, McGraw-Hill Book Co., New York, 4th edition, 1960, third printing (with limited revisions), p. 596 & pp. 605-607
(13) D. Marcuse, Engineering Quantum Electrodynamics; Harcourt, Brace and World, Inc.; New York, 1970, pp. 127-140. (A second augmented edition of this book, published in 1980 by Academic Press under the title Principles of Quantum Electronics, by Marcuse, also contains this material.)
(14) V. L. Streeter, Fluid Mechanics, McGraw-Hill Book Co., New York, 1960, p. 88
(15) J. R. D. Francis, A Textbook of Fluid Mechanics, Edward Arnold (Publishers) Ltd., London, 1958, Chapter 8; J. M. Kay, An Introduction to Fluid Mechanics and Heat Transfer, Cambridge University Press, Cambridge, 1963, Chapter 2; P. S. Barna, Fluid Mechanics for Engineers, third edition, Plenum Press New York, 1969, Chapter 6
(16) K. Karamcheti, A. B. Bauer, W. L. Shields, G. R. Stegen, and J. P. Woolley, National Aeronautics and Space Administration publication NASA SP-207, pp. 275-304 (esp. pp. 288, 295), (1969)
(17) K. Karamcheti and A. B. Bauer, SUDAER Rept. 162, Dept. of Aeronautics and Astronautics, Stanford Univ., (1963)
(18) A. Powell, J. Acoust. Soc. Am. 33, 395 (1961); 34, 163 (1962); many others
(19) W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)
(20) Young-Pil Kwon, J. Acoust. Soc. Am. 100 (5), November 1996, pp. 3028-3032; J. Acoust. Soc. Am. 104 (4), October 1998, pp. 2084-2089
(21) Y. Suematsu, Oscillating Phenomena of a Jet in a Jet-edge system; Bull of JSME, 251-15, p. 1479

Return to the Table of Contents

Return to The Home Page of Wade Blocker.



Comments will be appreciated.

wblocker@nmol.com

# THE HOME PAGE OF WADE BLOCKER

Comments will be welcomed.
wblocker@nmol.com

---

This page contains links to technical papers by Wade Blocker dealing with the application of the ordinary theory of electronic feedback oscillators to acoustic feedback oscillators. It is suggested that the papers be read in the order listed. To select a paper, left click on the the colored link.

---

## The theory of the edgetone oscillator

Size of file about 140 KB

---

## A theoretical model for the edgetone oscillator

Size of file about 9 KB

---

## The Development and History

## of

## "The Theory of the Edgetone Oscillator"

Size of file about 22 KB

---

## Distributed Circuit Oscillators

### Electronic Feedback Oscillators

**and**

**Acoustic Feedback Oscillators**

**Compared and Contrasted**

Size of file about 14 KB

---

## On the Pitch of Organ Pipes

**An Analysis of Data of Lord Rayleigh's**

Size of file about 30 KB

---

## The Edgetone Oscillator as The Exciting Agent

## for

## The Organ Flue Pipe

Size of file about 14 KB

---

## A THEORETICAL PREDICTION

### OF THE

# WEISENBERGER SCALING LAWS

### FOR THE

## ORGAN FLUE PIPE

Size of file about 16 KB

---

Comments will be welcomed.
wblocker@nmol.com

# A Theoretical Model

# for the

# Edgetone Oscillator


## Wade Blocker

wblocker@nmol.com



March, 2004


This paper presents and rationalizes a simple model for the edgetone oscillator, the development of which led to the paper "The theory of the edgetone oscillator." That paper is available as the lead paper on the web site where this paper is found. It is recommended that that paper be read in order to understand every point discussed here. This paper is being presented to clarify and further rationalize the model on which I based "The theory of the edgetone oscillator."

The edgetone oscillator ceased to be of interest to mainstream physicists over sixty years ago which is probably the reason it has remained unexplained. It appears that everyone now working in the field starts with the misconception that the phase velocity of the jet wave in the oscillator is a small fraction of the jet particle velocity because that is what everybody keeps on saying, parroting each other, a misconception that these workers seem very reluctant to give up. But a review of the origins of this erroneous belief shows that it is not justified by the available data, and shows in fact that it is seriously wrong. Part of the problem is that they don't define very well just what they mean when they state their belief. This problem is discussed in my paper.

The author of "The theory of the edgetone oscillator" has brought it to the attention of a few authors of books or papers that discuss the edgetone oscillator. It is the opinion of this author that none of these present a correct view of the relation between the phase velocity of the jet wave and the velocity of the jet particles. They don't even seem to recognize that the jet particles slow down markedly as they cross the aperture to edge gap in this oscillator. This makes it impossible for them to develop a successful theory of the edgetone oscillator.

None of these has openly challenged the validity of the application in my paper of Schlichting's equation for the slowing of the jet particles to Brown's experimental data, from which every conclusion of mine about the edgetone oscillator can be shown to follow without the interposition of any further theoretical considerations. Nevertheless a simple theory can be presented that still requires Schlichting's equation to complete the application of the theory but predicts a relation between the phase velocity of the jet wave and the velocity of the jet particles that is made independently of Schlichting's equation, but which prediction the application of Schlichting's equation to Brown's data verifies. It also predicts that the edgetone oscillator is a transit time oscillator, which prediction the application of Schlichting's equation to Brown's data also verifies.

The authors don't like the theory I present for the jet wave in the gap and challenge my conclusions on that basis, ignoring the fact that my conclusions don't depend upon the theory of the jet wave that I present. The conclusions can be and were drawn from my theory of the jet wave, but they are independent of that theory. As I point out in the paper under discussion they can be drawn just as well on a purely empirical basis from Schlichting's equation applied to the available experimental data, particularly to that of Brown. To challenge my conclusions requires that either Schlichting's equation or Brown's data, or both, be shown to be incorrect. This has not been done and presumably cannot be done.

The critical assumption of my theory of the jet particle motion in the gap is that the oscillator is a push-pull oscillator. Everything follows from that assumption. I assume that the edgetone oscillator is a push-pull oscillator because there is mirror symmetry about the x-z plane through the aperture and the edge. In the common electronic oscillators mirror symmetry in the circuitry results in a push-pull oscillator, so I assumed that mirror symmetry results in the same thing for the edgetone oscillator. That means the driving forces on the jet particles are effectively 180 degrees out of phase on the two sides of the stream of particles. My equations for the jet wave follow in straightforward fashion from that assumption. I don't have to justify this assumption *a priori*, the assumption has instead the *a posteriore* justification that it leads to predicted results that are in agreement with the experimental facts. Actually this how all theories in physics have been established, they have been postulated first and then justified by the agreement of their predictions with the experimental facts. It is not even necessary that the initial assumptions be correct. It is just necessary that they incorporate enough of the truth that they lead to valid conclusions. Some theorists discussing the edgetone or the organ flue pipe, professional physicists even, seem to forget this fundamental fact at times.

You don't have to have a mental picture of how things work in physics, although

of course people are much more comfortable about things when they do have such a picture. The most successful physical theory in existence is quantum mechanics, and there is no one in the world who has a mental picture of how it works, no one can claim to understand it, although there are innumerable people who know how to apply it. The results that quantum mechanics predicts are in agreement with the experimental facts and therefore quantum mechanics is accepted as a correct theory. No exceptions to its predictions have yet been found. The equations I present for the edgetone oscillator suffice to predict all the basic experimental results of operating an edgetone oscillator and by the same standards that apply to quantum mechanics you would have to say that I have a correct theory. There are no competing theories that can predict the experimental results. This is the first theory developed that can predict Brown's and Carriere's experimental results for the edgetone oscillator.

The edgetone oscillator is still in the area of classical physics so a mental picture of its operation is surely possible, but it is not necessary. However for the edgetone oscillator a qualitative picure or model of its operation is easily developed and a theory based on this qualitative picture will give quantitative predictions that are in agreement with all the available experimental data.

If the 180 degree phase difference postulated above and in my paper "The theory of the edgetone oscillator" actually exists then the particles in the jet stream in alternate half cycles of the edgetone oscillation should be directed in alternating puffs into the upper and lower halfspaces, that is directed alternately above and below the plane of mirror symmetry. Such an alteration in particle motion should cause a corresponding shift in where the center of the pressure wave is. The center of pressure should be alternately above and below the plane of symmetry. That shift in position justifies the idealization, or simplification if you will, that there are pressure variations on the the two sides of the stream of jet particles (that is, above and below the jet stream in my presentation) that are 180 degrees out of phase. The assumed pressure difference gives rise to a predicted particle motion and this predicted particle motion gives rise to the assumed pressure difference on the two sides of the jet stream. That means that the feedback loop exists which every oscillator with a continuous output requires. This picture does not have to be demonstrated *a priori* to be correct although one hopes that it is and indeed it appears that it might be, its real justification is *a posteriore*, that it leads to predictions in agreement with the experimental facts. Such agreement is the justification for the use of the assumptions.

The predictions deduced from the model are in complete agreement with the experimental facts so the assumptions of the model are justified.

Return to The Home Page of Wade Blocker.

Comments will be appreciated.

wblocker@nmol.com

# The Development and History

# of

# "The Theory of the Edgetone Oscillator"


## Wade Blocker

wblocker@nmol.com


September, 2003

It is recommended that the paper "The theory of the edgetone oscillator," available at the same web site as this present history of the paper, be read before this paper. That will make clear all the points discussed in the present paper.

This theory was developed in the years 1971 through 1973. A brief account of the work was presented at the Los Angeles meeting of the Acoustical Society of America, 30 October-2 November, 1973; and an abstract of that presentation was published in 1974 [W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)]. The key equations of the paper under discussion, equations 9, 11, 12, 13, and 24, are presented in the abstract. The abstract's statement that the phase velocity of the jet wave in stage one is twice the jet particle velocity there is incorrect. The error arose from the hasty assumption that only the second term of equation 20 of the paper was important for very small values of wT. (The theory predicts that the phase velocity of the jet wave at a point very near the aperture is four-thirds times the velocity of the jet particles at that point, and that for points not closely adjacent to the aperture the particle and phase velocities are for practical purposes equal.)

A first version of this paper was written in late 1971 and early 1972. The present version was written in 1974. Revisions were made in 1999 and 2000. The original paper stated only casually that the oscillation would be considered as a perturbation to the motion of the jet. A major purpose of the recent revisions was to emphasize that the theory being presented was a perturbation theory, and to show that this approach was fully justified. Text to that purpose has been added. No equations and no conclusions were changed. One new equation, equation 7,

was introduced. The literature since early 1974 has not been reviewed. A detailed account of the theory has not been previously published.

In mid 1971 the author got sufficiently interested in the flute and organ pipe to want to know how they worked (the author has a Ph.D. in Physics received in 1952 from the University of California in Berkeley where he worked under the direction of Professors Edwin M. MacMillan (a Nobel prize winner in Physics) and Wolfgang K. H. Panofsky in the late 1940s and early 1950s). A survey of the acoustics literature established that much is known about the operation of these musical instruments and that many books and thousands of papers had been devoted to them and people knew how to build excellent instruments, but that there were no theoretical papers which gave an adequate theory of their operation. It seemed to be established however that the flute and organ pipe were related to and somehow dependent upon the acoustic edgetone oscillator, which seems to be a drastically simplified version of these musical instruments incorporating some of their features but lacking the resonant acoustic air column which controls the frequency they produce.

A search of the literature devoted to the edgetone oscillator established that for this oscillator there were relatively few papers giving basic experimental data on its operation, the best of which were the papers of Brown (published in 1937), and Carriere (published in 1925), both referenced in the paper being discussed here. There were literally hundreds of theoretical papers by many authors trying to explain the operation of the edgetone oscillator and particularly the results of Brown.

Brown's experimental data is excellent, but there are serious problems in his presentation and interpretation of his data which were not recognized by these theorists. Dr. Alan Powell, to be mentioned again later, was and has been perhaps the most prominent of these, having published many letters and papers starting about 1953 devoted mostly to the elaboration of an approach and equation he had proposed in 1953 in his attempt to fit an empirical equation to Brown's data. Dr. Powell had accepted as correct Brown's faulty interpretation of his data. Although Powell did develop a large following among others who adopted his approach and equation in their own attempts to explain edgetones, his approach has never been successfully applied to predict Brown's basic data.

For some reason the equally important experimental results of Carriere have been almost universally ignored by theorists. None of their many papers has offered a theory that can be applied to predict Brown's experimental data or Carriere's.

In addition to the many papers published in the effort to explain the production of

edgetones, edgetones are briefly mentioned in many books devoted to explaining how common musical instruments, including the flute and organ flue pipe, work. Most of these books and papers and the discussions of the flute or organ pipe at least mention and some give great stress to the assertion that the phase velocity of the jet wave in the aperture to edge gap of the edgetone oscillator or of these musical instruments is only a small fraction of the velocity of the jet particles whose motion gives rise to the oscillations. A review of the literature shows that what the assertion or statement means is never clearly stated. A definition is necessary because the jet particles do not have a constant velocity all across the gap from jet aperture to edge in either the edgetone oscillator or the flute and organ pipe, the jet particles slow down. This slowing is very important for the edgetone oscillator, although of lesser importance usually for the flute and organ pipe. The simplest assumption is that the statement means that the jet particle velocity and the phase velocity of the jet wave in the gap are both constant across the gap and that the phase velocity is much less than the jet particle velocity. Another possible assumption is that the statement means that the phase velocity of the jet wave at a given position in the gap is a small fraction of the jet particle velocity at that same position in the gap, with nothing implied about the magnitude of the velocities at a given position. But actually the exact meaning of the statement is never expressed.

Apparently the first of these two options just given was tacitly assumed but not explicitly expressed in all prior attempts to explain Brown's results, and this was in fact tacitly assumed but not stated by Brown himself in presenting his results. But neither version of the interpretation of the statement is correct. Except very near the jet aperture the phase velocity of the jet wave at a point in the gap is almost equal to the jet particle velocity at that same point, this is established empirically by Schlichting's equations for the slowing of the jet particles and Brown's experimental data for the phase velocity of the jet wave near the edge, without the necessity of introducing any theory of the wave motion in the gap. Therefore neither the jet particle velocity nor the phase velocity of the jet wave is constant across across the gap, both velocities are for practical purposes equal to each other at a given point in the gap and both decrease as the gap is traversed. The major error in Dr. Powell's and other theorists' work has been their uncritical acceptance of the general belief, based upon their acceptance of Brown's erroneous interpretation of his data, that the phase velocity of the jet wave in the aperture to edge gap is a small fraction of the jet particle velocity in the aperture to edge gap. This belief resulted from their failure to to give an unambiguous definition of this statement.

It is evident that a clear understanding of the operation of the edgetone oscillator will not be achieved until the shibboleth about the phase velocity ingrained into

the minds of those studying the edgetone oscillator for the past seventy years and adopted in almost every paper or book discussing this oscillator is eliminated from the phenomenology assumed for the oscillator. This author is not aware any book or paper, other than his own, discussing the edgtone oscillator that presents a correct discussion of the relation between the phase velocity of the jet wave in the aperture to edge gap and the velocity of the jet particles in the gap. In general these books and papers exhibit no recognition even that the jet particles do not maintain their initial velocity just out of the aperture as they cross the gap.

To summarize, the phase velocity of the jet wave in the edgetone oscillator and in musical instruments such as the flute and organ flue pipe is not a small fraction of the jet particle velocity, as stated to be the fact in most texts and papers dealing with this oscillator and these musical instruments. To slightly adapt here a short but very appropriate quote from T. Needham written about a different subject but for a similar situation (T. Needham, Visual Complex Analysis, Oxford University Press, 1997, p. 386): *"We have made a fallacy of an assertion that is to be found in most texts. Perhaps the mere frequency with which this myth has been reiterated goes some way to explaining how it has acquired the status of fact."*

Brown was able to determine phase velocities only for stages two and higher and for these stages only just before the edge. Actually Brown measured the wavelength in the jet wave just before the edge. Multiplying this wavelength by the frequency observed gave the phase velocity of the jet wave just before the edge. The phase velocities Brown measured just before the edge were a small fraction of the jet particle velocities just out of the jet aperture. So Brown erroneously made the sweeping conclusion that the phase velocity of the jet wave was a small fraction of the jet particle velocity. Brown should have recognized that the jet particles slow down in crossing the aperture to edge gap and should have compared the phase velocity he measured near the edge to the jet particle velocity also near the edge, which would have been a more natural comparison. The phase velocities Brown had measured just before the edge were actually for practical purposes just the velocities to which the jet particles had slowed just before the edge but Brown had not recognized this, nor was this recognized by Powell and the many others trying to explain Brown's data, who had accepted as correct the main features of Brown's erroneous interpretation of his experimental data. The near equality of these two velocities is demonstrated in the paper under discussion here.

Since the phase velocity Brown found just before the edge was for practical purposes equal to the velocity of the jet particles just before the edge, it is strongly suggested that the phase velocity of the jet wave varies with position in the aperture to edge gap since the velocity of the jet particles varies with position in

the gap, the jet particles slow down as they cross the gap. This automatically means that the wavelength of the jet wave varies with position in the gap. But implicit in Brown's treatment and interpretation of his experimental data are the tacit assumptions that the wavelength is constant across the gap, and that the phase velocity and jet particle velocity are also constant across this gap.

Furthermore Brown had no experimental values whatever for wavelengths or phase velocities in stage one oscillations. But he inexplicably included values for stage one in his table of experimental values. A careful reading of his paper reveals that his values for stage one were guesses he had made. These entries should not have been included in his table but they were apparently accepted as genuine experimental data by later theorists. No one had pointed out that the entries were invalid.

Once it is acknowledged that the jet particles do slow down quickly as they cross the aperture to edge gap and the error about the jet wave phase velocity is eliminated from our mental picture of the phenomenology of the edgetone oscillator, it becomes rather an easy task to develop a theory of the oscillator, as demonstrated in my paper. The paper does predict every essential feature known about edgetone production then and now, including the prediction of the phase velocities Brown measured, which no one else as yet has been able to do. The basic data about edgetone production is best presented still in the papers of Brown and Carriere, and any acceptable theory of edgetones must predict their results.

It seemed to the author that surely this edgetone oscillator was just a simple feedback oscillator that could be explained in the same way that a multitude of electronic oscillators have been explained. My initial guess was that it might be an acoustic analogue of an electronic transit-time oscillator which is governed by the equation $fT = k$ where $f$ is the frequency of the oscillator, $T$ is the time required by electrons to cross a gap, and $k$ is a constant that depends upon the mode of oscillation. This electronic transit time oscillator has been carefully analyzed by Dietrich Marcuse (Dietrich Marcuse; Engineering Quantum Electrodynamics; Harcourt,Brace & World, Inc.; 1970; pp. 135-140). For the edgetone oscillator I thought $T$ would be the transit time of a jet particle from the jet aperture to the edge. A search of the literature showed that it would be easy to calculate this transit time for the edgetone oscillator but that apparently no one had ever investigated the possibility that the edgetone oscillator might be a simple transit time oscillator.

This was before the availability of small handheld electronic calculators, and the calculation of $T$ by hand for Brown's experiments although simple would be tedious, so I set up the equations now given in my paper for the calculation of $T$

with b, X, and v(0) as inputs and gave the equations and Brown's experimental data to a friend who had access to the big mainframe computer at the company where I worked and asked him to calculate the fT product for me for every entry in Brown's table of experimental data giving f as a function of the parameters b, X, v(0), and stage number in Brown's experiment. In about two weeks he provided me the data displayed in Table 1 of my paper "The theory of the edgetone oscillator." This table established empirically beyond any doubt that my guess was correct, the edgetone oscillator was indeed a transit time oscillator, governed by the simple equation fT = k where each operating mode of the oscillator has its own unique value of the parameter k. These computer calculations also established from the experimental data that the values of the constant k for the oscillator stages 1, 2, 3, 4, etc., would be very nearly k = 0.500, 1.250, 2.250, 3.250, etc. This equation obviously requires that the calculation of the transit time T take account of the slowing of the jet particles in crossing the gap from the jet aperture to the edge in those cases where the distance X is so great that a significant slowing of the jet particles has occurred. In Brown's experiments in many of his reported cases it can be shown that the jet particles had slowed to less than forty percent of their velocity just out of the jet aperture by the time T at which they reached the edge. Nevertheless Brown in his treatment and interpretation of his data had ignored the slowing of the jet particles, in effect tacitly assuming that the jet particles did not slow down.

By the time I had received the data of Table 1, I had developed equations 16 through 20 of my paper but had not yet seen how to interpret them to predict the fT values that would result in oscillations, although I was convinced that they could be so interpreted. I also saw that equation 8 of the paper for all practical purposes predicted as the particle velocities to which the jet particles had slowed at the edge distance just the phase velocities which Brown had measured for the jet wave just before the edge. Then I saw that the simple interpretation of equation 19 now offered in my paper would predict almost exactly the empirical fT values that had been developed from Brown's data and summarized as equation 15 of my paper. My equations also predicted that phase velocities measured near the edge should for all practical purposes equal the velocity to which the jet particles had slowed near the edge, and indeed the experimental data of Brown's indicated that this was true.

I was satisfied that I had now explained the basic features of the operation of the edgetone oscillator and had provided a means of calculating from theory without introducing any empirical factors everything that Brown had observed in his experiments. I could also explain what Carriere had observed, if one accepts the use of the empirical numerical factors commonly found necessary to correct the jet velocities calculated from Bernoulli's equation to the jet velocities actually

measured in experiments (Carriere gave the blowing pressure in his experiments rather than the initial velocity of the jet particles.) It had also turned out that no theory of the operation of the edgetone oscillator was really necessary. Brown's basic experimental data and the material in the literature on the slowing of the particles in fluid jets after they leave the jet aperture were all that was required in order to show that the edgetone oscillator was a transit-time oscillator governed by the equation $fT = k$ where $k$ depends upon the oscillation stage or mode.

I wrote all of this up in a paper very much shorter than the present expanded paper, including every equation 1 through 25 of the present paper but the introductory equation 7 which I put in later, and including Tables 1, 2, 3, 4, and 5 of the present paper. These tables show that my theory predicts with great accuracy everything that Brown and Carriere found in their experiments. I submitted the paper in late 1971 or very early in 1972 to the Journal of the Acoustical Society of America for consideration for publication. The Journal sent the paper to an employee of a Naval Research Laboratory in San Diego for a decision on publication, who sent it to an employee of another Naval Research Laboratory who was (presumably) an expert on the edgetone oscillator, actually to the Dr. Alan Powell mentioned above who was then head of the Navy's David Taylor Model Testing Basin in the Washington, D.C., area, and later in 1990-1991 president of the Acoustical Society of America, for review. Dr. Powell is evidently an accomplished administrator, although it is the opinion of the present author, that it has not been demonstrated that Dr. Powell is a sound scientist, his many years of effort devoted to attempts to explain the operation of the edgetone oscillator have not led to any significant increase in our understanding of the oscillator, and in fact the present author believes that his influence has been a hindrance to the development in the interested community of a good understanding of this oscillator.

Dr. Powell objected to the paper and recommended in the strongest terms that it not be published, stating that it was in conflict with the fact established in Brown's experiments that the phase velocity of the jet wave was only a small fraction of the jet particle velocity and not equal to or greater than the jet particle velocity as my paper would indicate (Apparently Dr. Powell had no objections to my being informed that he was the reviewer. I was informed of the identity of the reviewer and actually had the opportunity not only to see his comments but also later to speak to him.) It did no good to point out to Dr. Powell and the editor in San Diego that Brown had made egregious and incorrect assumptions in interpreting his data that made his conclusion about the phase velocity invalid, and that any paper on edgetones that predicted with great accuracy every aspect of Brown's and Carriere's data in exquisite detail, including the prediction of Brown's measured values of phase velocity, without the necessity of any empirical factor introduced

to force a fit to the data, ought to be published. My personal view at that time and now is that as soon as Brown's untenable and obviously incorrect assumptions had been pointed out, it should have been obvious to any qualified physicist that Brown's interpretation of his admittedly very good experimental data was wrong.

I spent almost two years until the end of 1973 trying to convince Dr. Powell and the San Diego editor that every conclusion of my paper about the edgetone oscillator depended only upon the validity of Brown's basic experimental data and the validity of the well known equations for the slowing of a turbulent jet, and was independent of the theory I offered for the jet wave in the gap. If Brown's data and Schlichting's equations for jet slowing were correct, and there was no reason to doubt their correctness, then every conclusion of mine about the edgetone oscillator was correct on a purely empirical basis, with no theory involved although I believed my theory was correct. I also told the editor in San Diego that I thought Dr. Powell would be embarrassed by and uncomfortable with my paper because it made nonsense of almost everything Dr. Powell had published on edgetones in the previous twenty years, and there was a lot, so it was clear that he might dislike my paper, even though it accomplished everything that Dr. Powell himself had been trying to do since at least his paper of 1953, that is, it presented a method to predict everything that Brown saw. But Dr. Powell's objections to the publication of the paper prevailed. Finally after a delay of almost two years, near the end of 1973, the San Diego editor told me he would not accept the paper.

Dr. Powell was a strong supporter of the statement that the phase velocity of the jet wave was a small fraction of the jet particle velocity, ill defined though that statement may be. The remarks in my present paper about the inadequacy of Dr. Powell's work on edgetones were added only after my paper had been rejected. I wanted to make clear what the conflicting theories were, what objections to my paper were made that resulted in its rejection, and why those objections were ill-founded. I am still unable to understand why a reputable scientist of Dr. Powell's standing would object to the publication of a paper that predicted with great accuracy every thing that Brown and Carriere had observed about the edgetone oscillator, as demonstrated by tables 2 through 5 of my paper and by the comparison of the phase velocities Brown observed near the edge to the jet particle velocities near the edge, when the goal of every scientist working on edgetones was to make those predictions. Dr. Powell himself in the many letters and papers he had published about edgetones was trying to make those predictions but had never been able to accomplish it.

Short papers presented at meetings of the Acoustical Society of America were not subject to review and could be presented by any qualified member of the Physical Society, so to get the work on record I gave a short summary of the work at the

Los Angeles meeting of the Acoustical Society of America held 30 October - 2 November, 1973. An abstract of the presentation as indicated above was published in 1974. The most important equations of this paper are included in the abstract [W. Blocker, J. Acoust. Soc. Am. 55, 458(A), (1974)].

Since this work was done as a hobby and had nothing to do with what I did for a living, and there appeared to be an insuperable obstacle (i.e., the objections of Dr. Powell) to getting it published in JASA, and also because it was occupying a large portion of my spare time, I decided to drop the matter. However I did prepare a greatly expanded version of the paper in early 1974 which presented the theory in more detail and pointed out in considerable detail my findings about the phase velocity of the jet wave, which was the area where I think Dr. Powell was most seriously in error in making his recommendation and the editor in accepting the recommendation.

I then put the paper aside for almost twenty five years, during which time I made no effort to follow the literature on edgetones. During those years apparently almost no progress was made in explaining the operation of the edgetone oscillator although many papers were still being published. There was still no published theory or even an empirical procedure that could predict the experimental data of Brown and of Carriere.

With the astounding development of the Internet in the 1990s, there was no bar to my publishing the paper myself on the Internet. So I decided after I got myself a modern computer and access to the Internet, that there were surely some people who would be interested in the paper. I made it available on the Internet in early 1997. A search of the Internet with the Google search engine using either "edgetones" or "organ flue pipes" as the search topic should turn up my paper. I have made a few minor changes of wording in editing the paper for the Internet and I added later the section of the paper treating the data of Dr. Suematsu on the edgetone, but the paper is largely as I wrote it in 1974.

It is still my opinion as a professional physicist, although now retired, that the Journal of the Acoustical Society of America was seriously wrong to reject the paper and Dr. Powell was certainly wrong to recommend the rejection and the editor in San Diego was wrong to accept Dr. Powell's recommendation. The paper does predict everything that either Brown or Carriere saw in their experiments on the edgetone oscillator, there are no other good summaries of the facts of edgetone oscillator operation, and ordinarily such a paper should be published. Indeed, looking at the extremely good accuracy with which the experimental results of both Brown and Carriere were predicted as shown in Tables 2 through 5 of the paper, I find it almost incredible that two presumably competent scientists would

decide that the paper was nonsense and should not be published. Dr. Powell's professed reason for his recommendation was clearly wrong since it was easily demonstrated that Brown's experiments did not show that the phase velocity of the jet wave was only a small fraction of the jet particle velocity.

It is surprising that more than sixty years after the publication of the papers of Brown and Carriere and the publication of literally hundreds of experimental and theoretical papers dealing with the edgetone oscillator, the papers of Brown and Carriere are still the best papers presenting experimental information on the fundamental properties of edgetone oscillators. Just beware of Brown's interpretation of his data. If you need quantitative information on how the frequency of the oscillator depends upon the physical parameters of the jet edge system and the initial velocity of the jet particles you still need to go to the papers of Brown and Carriere. Their basic experiments have not been repeated. Indeed it seems that mainstream physicists ceased to have much interest in these acoustic oscillators more than sixty years ago which I think is perhaps a major reason their operation has remained unexplained in the published literature. Another reason of course is that almost everyone working to explain the edgetone oscillator, such as Dr. Powell, seems imbued with the conviction that Brown's interpretation of his data is correct. Almost every book or paper the author has found that discusses the edgetone oscillator or the operation of flutes or organ pipes has the erroneous statement that the phase velocity of the jet wave is only a small fraction of the jet particle velocity; adding to the difficulty is that no exact meaning for this statement is ever given. Mainline physicists went from the discovery of the quantum to quantum mechanics in twenty five years, that is in one sixth of the one hundred fifty years since the discovery of the edgetone oscillator, but the edgetone oscillator is still not explained in the published scientific literature, except for the present paper under discussion which is found only on the Internet. The explanation of the operation of edgetone oscillator turns out to be simple. Its behaviour is quite analogous to the that of conceptually similar electronic oscillators.

Return to The Home Page of Wade Blocker.

Comments will be appreciated.

wblocker@nmol.com

# Distributed Circuit Oscillators

## Electronic Feedback Oscillators

## and

## Acoustic Feedback Oscillators

## Compared and Contrasted


### Wade Blocker

wblocker@nmol.com



November 2, 2000



**Abstract.**

It is pointed out that typical acoustic oscillators are closely analogous to very high frequency distributed circuit electronic oscillators using electron beams instead of electric currents confined to current conductors and cavity or transmission line resonators instead of lumped circuit RLC components. The proven principles and techniques of analyzing these electronic oscillators should be routinely applied to analyze the behavior of acoustic oscillators. For example, the analogy between the klystron oscillator and the edgetone and organ flue pipe oscillators provides the principles and techniques needed to analyze these acoustic oscillators.


**The Analysis of Acoustic Oscillators as Distributed Circuit Oscillators.**

It should perhaps be pointed out that any oscillator producing a continuous output is necessarily a feedback oscillator. The term "oscillator" in common usage frequently means an oscillator producing a continuous output and perhaps most persons use the term with this meaning, thereby implying a feedback oscillator, but the term "oscillator" is also applied in physics to designate such things as a struck rod or struck tuning fork which produce damped vibrations that die away. Some mechanism must exist to continuously replace the energy dissipated by the oscillator if the output is to be continuous. No oscillator exists that produces a

continuous output that is not a feedback oscillator. In this paper the word "oscillator" means a feedback oscillator, that is, an oscillator producing a continuous output. In the distributed circuit oscillator the feedback mechanism may not always be immediately obvious but it is inevitably there.

[As a matter of interest, many common items not commonly thought of as feedback oscillators can in fact be analyzed as such. For example, steam engines and automobile internal combustion engines have feedback mechanisms that control the timing and flow of energy to these engines to control their speed and power output, and therefore in a technical sense are actually feedback oscillators. An analysis of steam engines as feedback oscillators can be found on pages 559-582 of the book Theory of Oscillators by Russian authors A. A. Andronov, A. A. Vitt, and S. E. Khaikin, available in English translation from Dover Publications, New York.]

The analysis of the usual lumped circuit R-L-C electronic feedback oscillator is much simpler than the analysis of the usual acoustic feedback oscillator. That is because the functions required in any feedback oscillator such as power input, signal input, signal amplification, frequency selection and control, signal output, and provision of a path for feedback of output to input are performed in the lumped circuit electronic oscillator by physically separated components of the oscillator whose individual actions are usually easily analyzed and concatenated to describe the oscillator performance. On the contrary in almost any acoustic oscillator these functions are inextricably mixed and it is usually impossible to identify unique points in the oscillator where the functions are performed. The acoustic oscillator is not a lumped circuit oscillator. Instead the functions are distributed over space in the oscillator and more than one function is being performed in the same physical space. Such oscillators might properly be called "distributed circuit oscillators". In the edgetone oscillator and the organ flue pipe oscillator the functions of power input, signal input, signal amplification (the conversion of input power to acoustic output power), and feedback are all accomplished in the same physical space, that is, in the space between jet aperture and edge. The result is that analysis of the acoustic oscillator is much more difficult than the analysis of the usual electronic oscillator which is a "lumped circuit oscillator". Indeed satisfactory analyses of the operation of very many common acoustic oscillators have not been achieved. The organ flue pipe oscillator is one of these common oscillators whose operation has not been satisfactorily explained previously. By a satisfactory theory we mean a theory that gives accurate quantitative predictions of the performance of the oscillator given the parameters that define its physical structure and conditions of operation.

But there are electronic oscillators that are quite analogous to acoustic oscillators.

These are usually very high frequency electronic oscillators that are not built with lumped circuit RLC components but instead use transmission lines or tuned cavities as resonant circuits and electron beams instead of electric currents confined to current conductors. The common klystron oscillator is such an oscillator. In the klystron an electron beam with some velocity V and therefore some kinetic energy KE either does work against an oscillating electric field or the field does work against the beam. In the first case the electron beam gives up energy to the oscillating electric field whose strength increases (or reaches a constant value) and we have oscillations. In the second case the field loses energy to the electron beam and oscillations die away. Which of these two things happens depends upon the phase relation between the motion of the electrons in the beam and the phase of the electric field oscillations. The result is that for oscillations to occur only those velocities of the beam electrons giving rise to certain preferred values of transit time are allowed. It is easily demonstrated (Ref. 1) by calculating the transit time of the jet particles in the edgetone oscillator when oscillations are occurring that the edgetone oscillator exhibits the same behavior.

The resonant air column of the organ flue pipe is completely analogous to the resonant cavity of the klystron and the jet stream of the organ flue pipe is analogous to the electron beam of the klystron. Physically the organ flue pipe's construction is conceptually completely like that of the klystron. There is a region of space where the jet stream and the oscillating pressure field of the organ flue pipe oscillator occupy the same space (the space between the jet aperture and the edge) where the interaction between them takes place. The same is true of the electron beam and the oscillating electric field of the klystron. The analysis of the klystron by Marcuse (Ref. 2) is based upon just this behavior. It has always been recognized that electric and acoustic resonant cavities are closely analogous to each other and it is apparent that the same is true for these acoustic and electromagnetic oscillators. In fact if the edgetone oscillator is analyzed assuming this close analogy to the klystron the same equation is found for the edgetone oscillator that Marcuse found for the klystron oscillator (Refs. 1 & 2), with exception that the edgetone oscillator has one additional mode not discovered by Marcuse for the klystron oscillator.

Both oscillators are found to be transit-time oscillators governed by the equation fT = a(n) where f is the frequency observed, T is the transit time of a jet or beam particle, and a(n) is a constant depending upon the mode of oscillation excited. The first mode (for n = 1) of the edgetone oscillator has no corresponding mode in the klystron oscillator but every succeeding mode constant a(n) (n = 2 and higher) of the edgetone oscillator agrees in value with the values a(n) (n = 1 and higher) of the klystron oscillator. The frequency of the edgetone oscillator is typically in the low to very high audio range, and the frequency of the klystron oscillator is

typically in the range of hundreds to thousands of megaHerz but both are governed by the same basic process and the identical frequency equation with the same values for the constants a(n), except for the first mode of the edgetone oscillator. These statements are true for the edgetone oscillator and the frequencies of best performance of the klystron oscillator. The klystron because it has a resonant cavity acting to lock the frequency of the oscillator to a frequency near its resonant frequency operates in a range of frequencies near the resonant frequency of the cavity whereas no such locking mechanism based on a resonant acoustic circuit exists for the edgetone oscillator. Nevertheless the analogy pointed out exists and proceeding in accordance with the assumption that the analogy is valid we produce a frequency equation for the edgetone oscillator that is in complete agreement with available experimental data on edgetones. In fact we do not have to have a theory of the edgetone oscillator to produce the equation $fT = a(n)$. We can obtain this equation empirically by calculating the transit time T using for example Schlichting's equations (Refs. 1 & 3) for the slowing of the jet particles and multiplying the frequency f of the oscillator in each mode by the transit time T that gave rise to that frequency.

The edgetone oscillator does not have a resonant circuit controlling its frequency of oscillation, but the organ flue pipe oscillator does have such a resonant acoustic circuit. The organ flue pipe oscillator can be considered and analyzed as an edgetone oscillator to whose feedback loop a resonant acoustic circuit has been added which serves to lock the frequency of the edgetone oscillator to a frequency close to the frequency of the resonant acoustic circuit (for a discussion of how this is accomplished, see Ref. 4). This makes the construction and operation of the organ flue pipe oscillator closely analogous to that of the klystron oscillator. There is a difference however. The organ flue pipe oscillator as a musical instrument operates in a mode corresponding to mode 1 of the edgetone oscillator which mode does not exist for the klystron oscillator. This makes only a trivial difference in the analysis of the organ flue pipe. The method of analysis used for the edgetone oscillator when applied to the organ flue pipe oscillator predicts with complete success the empirical scaling laws discovered experimentally by Richard Weisenberger (Refs. 5 & 6) for the organ flue pipe.

One of the key assumptions in the successful treatment of the edgetone oscillator and the application of the same principles to the prediction of the Weisenberger Scaling Laws for the organ flue pipe was that the jet particle stream of the edgetone oscillator and of the organ flue pipe in crossing the gap distance between the jet aperture and the edge interacts with the oscillating pressure field in the aperture to edge gap of these oscillators in the same way the electron beam of the klystron oscillator interacts with oscillating electric field of the klystron oscillator. Depending upon the phase relation of the motion of the jet particles to the phase of

the pressure oscillations, the jet stream either adds energy to the oscillations of the pressure field or takes energy from them. In the first case the pressure oscillations are reinforced and sustained oscillations occur. In the second case the pressure oscillation field loses energy and any oscillations die away. For oscillations to occur, the structural parameters and operating conditions of the oscillators must be chosen so that the phase relations for the build up of oscillations exist. Imposing the appropriate boundary for oscillations to occur leads to the equation fT = a(n) for these oscillators and give the values for a(n).

It is evident that the most significant interactions in the edgetone oscillator and in the organ flue pipe oscillator occur in the gap between the jet aperture and the edge, and not at either the jet aperture or the edge, which is the typical assumption in many attempts to explain edgetone oscillations or organ flue pipe oscillations. In particular, there has been no need to invoke vortices produced at the jet aperture, which travel across the aperture to edge gap distance with about half the speed of the jet particles, and interact with the edge to produce in some ill-defined way the phenomenon of edgetone or organ flue pipe oscillations. Such a speculation has been very popular as a possible mechanism for these oscillations but it has remained that, a speculation. No one proposing this mechanism has in fact produced a theory that can make quantitative predictions in agreement with the available experimental data on these oscillators.

It is apparent that acoustic oscillators are what we might legitimately call "distributed circuit oscillators" in contrast to the electronic "lumped circuit oscillators" one usually encounters for low radio frequencies. However in the high frequency microwave region distributed circuit electronic oscillators are the norm and many of these oscillators are conceptually identical to common acoustic oscillators. The same principles and procedures that are successful in explaining the operation of these microwave oscillators will explain the operation of the acoustic oscillators. The interaction of the stream of jet particles with the oscillating acoustic field may be somewhat different from the interaction of the electron beam with an oscillating electric field, but that is a detail, the behaviors of the oscillators in all fundamentals are identical. The difficulties that have been experienced in the past in analyzing acoustic oscillators may be attributable in large part to the failures to recognize the close analogy between distributed circuit acoustic and electronic oscillators and to apply the proven principles and techniques of analyzing distributed circuit electronic oscillators to the analysis of distributed circuit acoustic oscillators.

The study of Marcuse's theory of the klystron oscillator is highly recommended for anyone trying to understand the organ flue pipe oscillator and also the simpler but quite similar edgetone oscillator.

## References

(1) W. Blocker, The Theory of the Edgetone Oscillator, on the author's Home Page on the Internet at URL http://www.nmol.com/users/wblocker/index.htm
(2) D. Marcuse, Engineering Quantum Electrodynamics; Harcourt, Brace and World, Inc.; New York, 1970, pp. 127-140. (A second augmented edition of this book, published in 1980 by Academic Press under the title Principles of Quantum Electronics, by Marcuse, also contains this material.)
(3) H. Schlichting, Boundary Layer Theory, McGraw-Hill Book Co., New York, 4th edition, 1960, third printing (with limited revisions), pp. 605-607
(4) W. Blocker, "On the Pitch of Organ Pipes", on the author's Home Page on the Internet at URL http://www.nmol.com/users/wblocker/index.htm
(5) R. J. Weisenberger, "Flue Pipe Acoustics", on the Internet at the address http://rjweisen.50megs.com
(6) W. Blocker, "A Theoretical Prediction of the Weisenberger Scaling Laws for the Organ Flue Pipe", on the author's Home Page on the Internet at the address http://www.nmol.com/users/wblocker/index.htm

Return to The Home Page of Wade Blocker.



Comments will be appreciated.

wblocker@nmol.com

# ON THE PITCH OF ORGAN PIPES

## An Analysis of Data of Lord Rayleigh's

1973
Revised October, 2000

Wade Blocker

Comments will be appreciated.
wblocker@nmol.com

ABSTRACT: Ordinary electronic feedback oscillators and acoustic feedback oscillators are close analogues of each other. The formalism for the analysis of ordinary electronic oscillators is adapted to the analysis of Lord Rayleigh's data on the pitch of an organ pipe acoustic oscillator. A close fit is achieved between theory and Lord Rayleigh's data giving the change in pitch of an organ pipe as a function of the blowing pressure.

## Introduction

Ordinary electronic oscillators and acoustic oscillators are close analogues of each other and the same basic principles or formalisms that explain the operation of the one type of oscillator will also suffice to explain the operation of the other. In the present paper equations taken from the theory of electronic oscillators are directly applied to explain certain features of organ pipe operation.

Lord Rayleigh in 1882 (Ref. 1) in a paper entitled "On the Pitch of Organ-Pipes" published data giving the change in pitch of an organ pipe as the blowing pressure was changed. (I will use the word "pitch" rather than "frequency" on occasion because that is the word Lord Rayleigh used.) The author knows of no prior published treatment of Lord Rayleigh's data.

Lord Rayleigh determined the passive resonance frequency when unblown of a particular organ pipe about two feet long to be 255 Hz, which he assumed to be the natural resonance frequency of the pipe. When the pipe was blown with a blowing pressure of 0.8 inches of water the pipe sounded with the pitch 255 Hz. As the blowing pressure was increased above 0.8 to 4.2 inches of water the pitch sounded increased by 11 Hz, and as the blowing pressure was decreased below 0.8

to 0.39 inches of water the pitch sounded decreased by 4.0 Hz, with intermediate pitch changes for intermediate pressure changes. The complete set of data will be exhibited below. Lord Rayleigh offered no theory to explain the changes in pitch.

## A Theory of Organ Pipe Operation

Feedback oscillators in general are well understood. Any amplifier can be made into an oscillator with continuous output. Introduce a signal path from the output of the amplifier back to its input, so that the amplifier output is fed back to provide the amplifier's input, and oscillations can be produced. There are two conditions that must be satisfied for a continuous output at constant amplitude to result. First, the net signal gain around the feedback loop, taking account of both gains and losses, must be equal to one, exactly; and second, the change incurred in the phase of the signal in going around the loop, from input to output back to input, must be either zero or a multiple of two pi, so that the fedback signal is exactly in phase with the input signal. The oscillator automatically picks a frequency satisfying the phase condition. The amplitude of oscillation automatically adjusts itself until the loop gain is equal to one (the process by which that is accomplished will not be discussed here). If a loop gain as high as one cannot be achieved at the frequency, no sustained oscillation results. The facts just presented are probably already known to most readers, but I call attention to the two conditions because the second condition is the key to explaining Lord Rayleigh's data. These two conditions apply to any feedback oscillator, whether it be an acoustic oscillator such as a policeman's whistle or the radio frequency electronic oscillator in a cellular phone.

If there is a circuit element in the signal loop with an amplitude response to the signal sharply peaked at one frequency, that component usually takes charge of the frequency determination if the gain condition can be satisfied at that frequency. Both gain and phase conditions around the loop still have to be simultaneously satisfied. I shall assume henceforth that the gain condition is satisfied and discuss only the phase shift requirement around the loop which determines the frequency.

(Because I cannot write all equations in standard mathematical form on my computer, I will write most equations as they would appear if written and intended to be executed in a computer program. In particular I will use the symbol * to indicate multiplication, the symbol ^ to indicate exponentiation, and SQRT(X) to indicate that a square root is to be taken, in this case the square root of X is indicated. The standard order of precedence in evaluating mathematical operations is assumed. Also I will use the word "pi" in equations instead of the usual Greek lower case letter for pi. I will use /Z/ to mean the absolute value or magnitude of

Z.)

The input impedance Z of an electrical circuit consisting of a resistance R, an inductance L, and a capacitance C, all in parallel, to a signal of frequency f, is given by

```
Z = 1/[1/R + j*2*pi*f*C - 1/
(j*2*pi*f*L)]                                  (1)
```

If we define the resonant frequency fr = 1/[2*pi*SQRT(LC)] and the sharpness-of-resonance factor Q = R/SQRT(L/C), the impedance relation becomes

```
Z = R/[1 + jQ(f/fr - fr/
f)]                                              (2)
```

The column of air in an organ pipe is the acoustical equivalent of a tuned electrical circuit. The response of a sharply tuned circuit element, whether electrical or acoustical, is characterized by its impedance Z, again either electrical or acoustical. The absolute value of Z, or the magnitude of the response of the circuit above to a signal of frequency f is given by

```
/Z/ = R / SQRT{1 + [Q*(f/fr - fr/f)]
^2}                       (3)
```

where

/Z/ = the absolute value of Z, or the magnitude of the circuit response for the frequency f

R = the magnitude of the circuit response for the frequency fr

fr = the resonant frequency of the circuit element, that frequency at which the response of the element to a signal is the maximum

f = the frequency of the signal actually being applied to the element

Q = the "quality factor" defining the sharpness of the circuit response; the higher the value of Q, the more sharply tuned the circuit response. Q is a positive quantity.

The tuned circuit element is also characterized by the phase change it introduces

into the signal if the frequency is anything other than the resonant frequency of the circuit. This phase change must be taken account of in summing the total phase shift in going around the feedback loop. The phase shift is given by the formula

$$\tan A = -Q*[(f/fr) - (fr/f)] \quad (4)$$

A = the angle by which the phase is changed by the resonant circuit. At the resonant frequency, the angle A is zero; i.e., there is no phase change. At any other frequency, there is a phase shift defined by this equation. A positive frequency change results in a negative phase change. The higher the Q, the greater the magnitude of the change in phase for a given frequency change.

The selective tuned circuit and feedback loop are usually designed so that the desired operating frequency equals or is very near the resonant frequency of the frequency determining tuned circuit, but this is not strictly necessary; the circuit can oscillate anyway, all that is necessary if that the total phase change around the loop be zero or a multiple of 2*pi, and that the loop gain be equal to one at the operating frequency. Assume that the oscillator is operating, that the phase change around the loop is zero, and that the angle A is zero. Any multiple of 2*pi could be initially assumed for the phase change around the loop, but the assumption of zero phase change simplifies the analysis, and the possibility of some other multiple of 2*pi occurring will automatically reintroduce itself later. The angle A has to be between -pi/2 and +pi/2. The assumption that A = 0 is not necessary but it does simplify the analysis slightly. The oscillator is then operating at the frequency f = fr. If a phase change B is then introduced at some point in the loop other than at the tuned circuit, and the oscillator remains in operation but at a different frequency, what has happened is that the oscillator has shifted from the frequency fr to a frequency f such that the resulting phase shift angle A at the circuit is the negative of the angle B; the total phase change around the loop is still zero. Therefore if we can calculate or otherwise know what phase change A is made at the tuned circuit by some change in operating conditions, then we also know that a phase change -A caused somewhere else in the feedback loop by the same change in operating conditions is being compensated for. If we know the Q of the tuned circuit, we can indeed calculate the phase change A since the frequency change df brought about by any change in operating conditions is easily measured. This compensatory action of the tuned circuit provides us with a diagnostic technique for investigating the mechanism of organ pipe oscillations that does not appear to have been exploited.

There is an assumption frequently made in applying equation 4 that greatly

simplifies manual computations but not introduce significant error. If the the change in frequency is a small percentage of the frequency fr then equation 4 can be quite accurately approximated by

```
tan A = -2*Q*(df/
fr)                                          (5)
```

where df = the change in frequency = (f - fr); df is positive for an increase in frequency and negative for a decrease in frequency. As long as df is only a small percentage of fr, equation 5 is an adequate approximation to equation 4. In this day of ubiquitous electronic calculators and computers it is hardly necessary to approximate equations as simple as equation 4 so equation 5 was not used in the results presented here. However the author has done the calculations of this paper both ways and the results differ only trivially as expected.

If we knew the Q-factor of Lord Rayleigh's flue pipe, since we do know the resonant frequency of the pipe and the change in frequency with changes of blowing pressure, we could use equation 4 to calculate the change B in phase somewhere in the feedback loop that is caused by the change in blowing pressure and is compensated by the opposite change A in phase at the resonant air column of the pipe. The phase change A can be calculated by equation 4, so we set B = -A. Knowing B as a function of blowing pressure, we might be able to deduce something about the operation of the flue pipe. Actually what we are going to do, since Lord Rayleigh's data gives us the resonant frequency fr and f, and we donâ€™t know the Q of his organ pipe, is to solve for the Q of Lord Rayleigh's organ pipe instead, and see if a single constant and reasonable value results. Equation 4 becomes

```
Q = -[tan(-B)]/[(f/fr) - (fr/f)] = tan(B)/[(f/fr) -
(fr/f)]                 (6)
```

The equations describing oscillator operation are universal. The equations 2 through 6 above apply as equally to organ pipe oscillators as to electronic oscillators. To apply equation 6 to Lord Rayleigh's data, we have to supply a value for the phase shift B introduced by changing the blowing pressure from the blowing pressure which caused to pipe to sound at the pitch fr to a new blowing pressure causing the pipe to sound at a new pitch f. I think it is possible to make guesses at this angle B and to verify the reasonableness of the guesses by the consistency and reasonableness of the values obtained for Q, one value being obtained for each of Lord Rayleigh's blowing pressures.

Let us now assume that when the pipe is blown at the pressure that results in the

pipe sounding at its resonant frequency, the phase change contributed to the total phase change around the loop by the jet stream and its interaction with the pipe is some angle C, which we will measure in radians. We expect C to be a phase delay, that is, to be a negative angle. When the pipe is blown at a higher velocity than the velocity vr producing the frequency fr, it seems reasonable that the magnitude of the phase change should be less, as a guess less in inverse proportion to the blowing velocity. Therefore at a velocity vf producing the new frequency f, I am guessing that the phase shift C changes to the phase shift [(vr/vf)*C]. The change in the phase shift is therefore the angle {[(vr/vf)-1]*C}. This change corresponds to the angle B discussed above. We substitute this value of B into equation 6 above. Therefore we have guessed that the Q of Lord Rayleigh's organ pipe is

Q = [tan{[(vr/vf)-1]*C}] / [(f/fr) - (fr/f)]     (7)

We now have to assign a value to the angle C. We expect C to be a negative angle, or a phase delay. Evidently an angle of magnitude C equal to zero is too small, since there would be no phase shifts at all in that case. Also, the magnitude of an angle C equal to -pi is too large, since the value vf = 2*vr would then predict an infinite value of Q for a finite frequency change. After a little thought we conclude that the only value of the angle C for which equation 7 makes physical sense is C equal to -pi/2. For example, equation 4 tells us that for any finite value of Q the angle A equals -pi/2 when f equals infinity, equals zero when f equals fr, and equals +pi/2 when f equals zero. We might expect these values of f to correspond to velocity values vf of infinity, vr, and zero, respectively, and indeed these velocities would give the frequencies indicated. However it happens that because of the control the Q of the resonant acoustic circuit exerts on the frequency of oscillation many other velocities less than vr but greater than zero can also result in the angle A equal to +pi/2 and zero frequency. Since the angle B should be the negative of the angle A for these cases, this indicates that a value for the angle C of -pi/2 would predict the first two values for A for vf equal to infinity and vr respectively, and vf equal to vr/2 would predict the third value. Therefore we make the trial

B = [1-(vr/vf)]*(pi/2)     (8)

With this substitution, equation 7 becomes

Q = [tan{[1-(vr/vf)]*(pi/2)}]/[(f/fr) - (fr/f)]     (9)

As we expect, the angle change B is positive when the velocity vf is greater than the velocity vr. Equation 8 is in complete qualitative agreement with equation 4. As the velocity vf increases above the velocity vr, the frequency f of the flue pipe increases. As the frequency increases from zero to infinity, equation 4 shows that the phase shift A goes from +pi/2 to -pi/2, so the phase B, which is equal to -A, should vary from -pi/2 to +pi/2, which is predicted by equation 8 (B is equal to -pi/2 when vf equals vr/2, and B equals +pi/2 when vf goes to infinity). Equation 4 shows that the phase shift A varies between +pi/2 and -pi/2 as the frequency f varies from zero to infinity. Outside these bounds on A no oscillations occur. As vf decreases from infinity to vr/2 the phase shift B decreases from +pi/2 to -pi/2 which is within the range for which the possible variation of the phase shift A can compensate, so oscillations can occur if the gain condition around the feedback loop is satisfied. When vf equals vr exact resonance is predicted to occur. When B equals -pi/2 in equation 8 then A, which equals -B, is at the limit of the range within which it can correct for B and oscillations necessarily cease at that point as vf decreases further. Actually oscillations would have ceased at some value of vf larger than vr/2 because as equation 3 shows the gain of the oscillator would have dropped below the value necessary to sustain oscillations at some higher point. As vf decreases further equation 8 predicts that there will be a range of velocities vf between vr/2 and vr/4 where B is between -pi/2 and -3*pi/2 and therefore outside the range of values of B for which A can compensate, so the phase condition around the feedback loop cannot be satisfied and oscillations cannot occur.

As vf decreases still further equation 8 predicts that velocity regions where oscillations can not occur will continue to alternate with velocity regions where oscillations can occur. This is in fact the behavior that Lord Rayleigh saw in his experiment. Unfortunately his data in the low velocity region where he saw this behavior is qualitative only, so no quantitative comparison can be made in this velocity region between our theory and his observations. We should not expect too much from equation 8 so it is unlikely that it will predict with good accuracy the bounds of all these velocity ranges. However we will show that the equation does provide a very good empirical fit to Lord Rayleigh's data.

The significant parameter in organ pipe operation is the blowing velocity and not the blowing pressure. But pressure is very easily measured while determining velocity is more complex, so pressure rather than velocity is the parameter often given. Ordinarily Lord Rayleigh's given values of blowing pressures would have to be converted to blowing velocities. Blowing pressure is usually converted to blowing velocity by the use of Bernoulli's equation. This is fraught with considerable uncertainty. It has been shown theoretically that the velocity from a Borda tube aperture is 50 percent of what Bernoulli's equation predicts, from a sharp edged orifice it is found experimentally that the velocity is about 63 percent

of the Bernoulli prediction, and for a Venturi orifice the velocity can be close to 100 percent of the Bernoulli value (Refs. 2). This is a reflection of the fact that the idealized conditions assumed in Bernoulli's equation do not always or even usually exist in practical situations. Fortunately, for the calculations to be done here, only the ratio of blowing velocities is involved so any conversion factor cancels in using Bernoulli's equation to compute velocity ratios. This just happens to be a useful coincidence in our case, for most purposes we need the blowing velocity, and it is hard to get an accurate velocity given only the blowing pressure. Because we know from Bernoulli's equation that the ratio (vr/vf) has the same value as the square root of the ratio (pr/pf), we can write the last equation as

```
Q = tan{[1-SQRT(pr/pf)]*(pi/2)} / [(f/fr) - (fr/
f)]                        (10)
```

where

pr = that blowing pressure that results in the frequency fr

and

pf = that blowing pressure that results in the frequency f

Everything in equation 10 is standard electronic oscillator theory applied to the organ pipe, except for the single assumption that the interaction of the wind stream with the organ pipe when the blowing velocity is changed from vr to vf introduces a phase shift B that is equal to [1-(vr/vf)*(pi/2)], which means that at the resonant frequency fr this phase shift B is zero. A test of equation 10 is really a test of whether this assumed phase shift exists and whether its magnitude and velocity dependence have been properly estimated. We know from the fact that the frequency sounded changes with the blowing pressure that some phase change is introduced somewhere in the feedback loop other than just at the resonant air column by the change in blowing pressure. If we knew the Q of Lord Rayleigh's organ pipe, we could easily calculate this phase change and perhaps assign a locale to it.

Equation 10 is just a convenient equivalent of equation 9. Any units for pressure and frequency can be used in equation 10 since pressures and frequencies enter only as ratios. Lord Rayleigh gave the blowing pressure in inches of water. Remember that pr is 0.80 inches of water, and fr is 255 Hz.

Table 1 shows Lord Rayleigh's basic data and the application of equation 10 to his

data. Column 1 contains the blowing pressures pf applied to the pipe by Lord Rayleigh and column 3 contains the frequency change (f-fr) from the resonant frequency of the pipe. pf is the blowing pressure producing the change (f-fr) in the frequency sounded by the pipe. The quantities actually measured by Lord Rayleigh were pr, fr, pf, and the frequency differences from fr, i.e., he measured fr and (f-fr) by determining beat frequencies comparing the pipe's oscillating frequencies to a standard frequency of 255 Hz which matched the resonant frequency of his flue pipe. Column 2 contains the corresponding values of Q calculated using equation 10. Column 4 contains the theoretical values of frequency shift (f-fr) calculated from equation 10, assuming that Q has its average value 11.98, whose calculation is described below. The two values of (f-fr) in adjacent columns allow an easy and immediate comparison of experiment and theory. Column 5 gives the ratio of the calculated theoretical values of (f-fr) in column 4 to the experimental values of (f-fr) in column 3. Column 5 is also the ratio of Q in column 2 to 11.98 which is the average value of Q. Column 6 gives the frequencies of oscillation observed by Lord Rayleigh. Column 7 gives the frequencies this theory predicts assuming the value 11.98 for the Q of the organ pipe. Column 8, the last column, gives the ratio of the theoretically calculated frequencies to the experimentally observed frequencies. It is seen that the predictions of the frequencies observed by Lord Rayleigh are extraordinarily accurate. Except for the entries about which Lord Rayleigh expressed reservations, no error of prediction exceeded 0.3 percent.

---

Table 1. The application of equation 10 to Lord Rayleigh's data.
Blowing pressure, pf, is given in inches of water. Frequency differences, (f-fr), are given in Hertz. In the table "expt." stands for "experiment".

| pf expt. | Q theory | (f - fr) expt. | (f - fr) theory | [(f-fr)/(f-fr)] [theory/expt.] | f expt. | f theory | f,theory/f,expt |
|---|---|---|---|---|---|---|---|
| 4.20 | 14.47 | +11.0 | +13.34 | 1.21 | 266.00 | 268.34 | 1.009 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2.72 | 12.21 | + 9.3 | + 9.48 | 1.02 | 264.30 | 264.48 | 1.001 |
| 2.26 | 11.39 | + 8.4 | + 7.98 | 0.95 | 263.40 | 262.98 | 0.998 |
| 1.86 | 10.93 | + 7.1 | + 6.47 | 0.91 | 262.10 | 261.47 | 0.998 |
| 1.53 | 10.69 | + 5.6 | + 4.99 | 0.89 | 260.60 | 259.99 | 0.998 |
| 1.32 | 11.10 | + 4.2 | + 3.89 | 0.93 | 259.20 | 258.89 | 0.999 |
| 1.06 | 12.75 | + 2.1 | + 2.24 | 1.07 | 257.10 | 257.24 | 1.001 |
| 0.88 | 6.24# | + 1.5# | + 0.78# | 0.52# | 256.50# | 255.78# | 0.997# |
| 0.82 | 24.58# | + 0.1# | + 0.21# | 2.06# | 255.10# | 255.21# | 1.000# |
| 0.80 | ---- | 0.0 | 0.00 | --- | 255.00 | --- | --- |
| 0.75 | 13.13 | - 0.5 | - 0.55 | 1.10 | 254.50 | 254.45 | 1.000 |
| 0.68 | 14.18 | - 1.2 | - 1.42 | 1.18 | 253.80 | 253.58 | 0.999 |
| 0.64 | 10.35 | - 2.3 | - 1.99 | 0.87 | 252.70 | 253.01 | 1.001 |
| 0.57 | 9.68 | - 3.9 | - 3.16 | 0.81 | 251.10 | 251.84 | 1.003 |
| 0.53 | 12.84 | - 3.7 | - 3.96 | 1.07 | 251.30 | 251.04 | 0.999 |
| 0.48 | 20.10# | - 3.1# | - 5.18# | 1.67# | 251.90# | 249.82# | 0.992# |
| 0.39 | 25.52# | - 4.0# | - 8.44# | 2.11# | 251.00# | 246.56# | 0.982# |

---

The average value of Q equals 11.98, which I use in calculating the theoretical values for frequency shifts (f-fr) in column 4 of Table 1. Values of Q marked with the symbol # have been omitted in calculating this average value of Q. In computing the average of a set of values, it is standard practice and considered justifiable to discard as possibly spurious any values differing so very greatly from the mean of the other values. These doubtful data entries occur just where we might expect them. According to Lord Rayleigh's statement, slow beats, indicating small values of (f-fr), are "embarrassing" and difficult to determine accurately.

Also he states that at his lowest blowing pressures the measurements were difficult and uncertain. The data points we have excluded from consideration are exactly those to which Lord Rayleigh's remarks apply. Including the discarded values would change the average value of Q to 13.76 which is still an acceptable value for a power oscillator.

Unfortunately, we don't know the Q of Lord Rayleigh's organ pipe. Except for the four values of Q discarded, all the values of Q are close to the average value of 11.98, as shown by column 5 which is also the ratio of Q in column 2 to 11.98 which is the average value of Q, and therefore the procedure used results in a consistent value of Q. The value 12 is a very frequently used value of Q for the plate circuit of final amplifiers in radio-frequency transmitters. This choice results in adequate spectral purity of the transmitter output, adequate efficiency in converting input energy into output energy, and adequate ease of adjustments and transmitter loading. Exactly the same considerations should apply to the organ pipe oscillator, so a Q of 12 for the resonant column of an organ pipe is not at all surprising, and a value near that might indeed be expected *a priori*. At the present moment (i.e., in 1973) I have no values available of the Qs of actual organ pipes for comparison with this value. The calculations just done show that even modest values of Q have a very large effect in stabilizing the frequency of the flue pipe oscillator. For the edgetone oscillator the pressure changes in Lord Rayleigh's experiment would have resulted in the frequency being more than doubled, instead of increasing by only about four percent.

{[Note added in 1999]:John W. Coltman, in a paper "Jet drive mechanisms in edge tones and organ pipes", J. Acoust. Soc. Am., Vol. 60, No. 3, September 1976, pp. 725-733, refers to an organ pipe with Q-factors of 10.7, 17.3, and 16.7, which are sufficiently near to the value calculated for Lord Rayleigh's organ pipe to give some confidence in the procedure used.}

A repetition of such measurements as Lord Rayleigh's with the Q of the organ pipe used also being measured would be a very useful contribution to our store of experimental information on organ pipes. It would be better if blowing velocities directly determined rather than blowing pressures were given. Whether equations 7, 8, 9, and 10 of the theory just presented are correct in detail or not, equations 2 through 6 are known to be correct for electronic oscillators and should be correct for the organ pipe oscillator. Therefore, the availability of Q in addition to the type of information given by Lord Rayleigh would allow an experimental determination as a function of the blowing pressure or blowing velocity of the phase shift angle that I have called angle B. Such information would be very useful in understanding organ pipe operation and would allow the assumption of the present theory that B={[1-(vr/vf)]*[pi/2]} to be checked, or would provide

data allowing a better theory to be developed.

The values predicted, as the blowing velocity vf is steadily reduced below vr toward zero, for the angle B, B = [1-(vr/vf)]*(pi/2), suggest that velocity regions where oscillation is possible will alternate with velocity regions where oscillation is not possible. The successive regions where oscillation is possible evidently are successive velocity regimes where the phase delays going around the feedback loop are successive multiples of 2*pi, thus satisfying the phase condition necessary for oscillations to be possible. The gain condition would still have to be satisfied also. Lord Rayleigh's detailed data does not extend below vr (or pr) to the point where such behavior would be evident, so this check on the theory presented is not possible. However he does present qualitative data that shows that, as the blowing pressure is sufficiently reduced below pr, the alternating regimes predicted by our theory where the pipe sounds and does not sound do occur.

The theory just presented agrees very well with Lord Rayleigh's data, except where the data itself is suspect. One such example of course does not prove a theory, but it does suggest that the approach used is worth further investigation. If the value 12 is indeed a typical value of Q for organ pipes, as it might well be, then the presumption that the theory is correct is very strongly supported.

References

(1) Lord Rayleigh, On the Pitch of Organ-Pipes, Phil. Mag. XIII, 340-347, (1882); Scientific Papers 2, 95
(2) J. R. D. Francis, A Textbook of Fluid Mechanics, Edward Arnold (Publishers) Ltd., London, 1958, Chapter 8: J. M. Kay, An Introduction to Fluid Mechanics and Heat Transfer, Cambridge University Press, Cambridge, 1963, Chapter 2; P. S. Barna, Fluid Mechanics for Engineers, third edition, Plenum Press, New York, 1969, Chapter 6



To return to the Home Page, click on Home Page.

Comments will be appreciated.
wblocker@nmol.com

# THE EDGETONE OSCILLATOR

## AS THE EXCITING AGENT FOR THE

# ORGAN FLUE PIPE

December, 1998

Wade Blocker

Comments will be appreciated
wblocker@nmol.com

ABSTRACT: A connection between edgetone oscillations and the organ flue pipe acoustic oscillator is postulated, and a prediction made of what blowing velocities cause the flue pipe to sound, either at its fundamental resonant frequency or at some integer multiple of that frequency. There are at present not sufficient experimental data available to test or verify this prediction.

TABLE OF CONTENTS:



## Introduction

Edgetone oscillations have been explained in a previous paper (Ref. 1), and the variation in pitch of an organ flue pipe acoustic oscillator as the blowing pressure (or better, blowing velocity) is changed has been explained in a second paper (Ref. 2). Relying upon the close analogy between electronic and acoustic resonators, the application of principles from the theory of electronic oscillators allows numerical predictions to be made which are in very close agreement with the available experimental data for these acoustic oscillators. These papers give a diagram of the edgetone oscillator and define more fully most of the terms used in this paper. It is suggested that these papers be read before the present paper.

# The Edgetone Oscillator

It has been shown previously (Ref. 1) that the frequencies f observed in the edgetone oscillator are predicted by the following theoretical equation, which has been verified by comparison with the available experimental data.

$$fT(n) = a(n) \quad (1)$$

where

$$n = 1, 2, 3, 4, ... \quad (2)$$

The parameter n is the stage number (or mode number) of the edgetone oscillation that is occurring.

T(n) is the transit time of a jet particle from the jet aperture to the edge in mode n of the edgetone oscillator.

a(n) is defined by the theoretical sequence of numbers, derived in Reference 1,

$$a(n) = 0.500, 1.230, 2.239, 3.242, 4.244, 5.245, ... \quad (3)$$

for mode numbers n equal to 1, 2, 3, 4, 5, 6, ...

For later use we write equation 1 in the form

$$T(n) = a(n)/f \quad (4)$$

where T(n) is now the transit time required if we are to obtain the edgetone frequency f in edgetone oscillator mode n.

It was also shown in Ref. 1 that for a turbulent plane jet the transit time of a jet particle from the jet aperture to the distance x from the aperture is given by

$$T(x) = T0*\{[1 + (x/x0)]^{(3/2)} - 1\} \quad (5)$$

$$T0 = (2/3)*[x0/v] \quad (6)$$

$$x0 = 5.75b \quad (7)$$

b = the aperture slit width, and

v = the the initial jet particle velocity at the jet aperture.

It is seen that the transit time T(x) can be expressed in the form

$$T(x) = G/v \tag{8}$$

$$G = [(2/3)*x0]*\{[1 + (x/x0)]^{(3/2)} - 1\} \tag{9}$$

G has the dimensions of distance, depends solely upon the geometry of the jet-edge system, and is thus the same for all oscillation modes.

Equating the values of T from equations 4 and 8, we obtain

$$a(n) / f = G / v \tag{10}$$

$$v(n) = (G*f) / a(n) \tag{11}$$

v(n) is the initial jet velocity required, just out of the aperture, to give the frequency f in mode n of the edgetone oscillator.

## The Flue Pipe Oscillator

We now turn to the organ flue pipe. Assume that the passive resonant frequency of the flue pipe in its fundamental first harmonic mode when unblown is f. Assume that the parameters of the pipe's jet-edge system exactly match the parameters of the jet-edge system of the corresponding edgetone oscillator, that is, the aperture slit-width b, the aperture to edge gap width X, and the initial jet velocity v of the flue pipe sounding the frequency f are the same as those of the edgetone oscillator whose frequency in mode 1 is the same as the pipe's passive resonant frequency f. These terms are all fully explained in the cited references. Therefore when blown with the velocity v, the pipe should sound with the frequency f. The action of the resonant air column of the flue pipe is to act as a resonant circuit to select the edgetone frequency f, and the action of the blown flue pipe is to act as an amplifier to emphasize and amplify that selected frequency, and also to lock the frequency of the pipe to a frequency near f as the blowing velocity departs from

the value of v at exact resonance. The action of the complete flue pipe system is exactly analogous to the operation of the ordinary electronic oscillator, as set forth in Reference 2. In fact, the same equations that describe the operation of the electronic oscillator, when expressed in the proper generalized parameters, describe the operation of the flue pipe oscillator.

When blown with a velocity higher or lower than v, the pipe's frequency should vary as described in the paper of Ref. 2, "On the Pitch of Organ Pipes," which analyzes data published by Lord Rayleigh (Ref. 3) in 1882 showing the variation in the pitch sounded by an organ pipe as the blowing pressure was varied. As the blowing pressure, and consequently the blowing velocity, is lowered, the frequency sounded drops in frequency, and at some point either the loop gain condition or the phase condition defined in Ref. 2 as necessary for oscillation to occur cannot be maintained, and oscillation ceases, that is, the pipe ceases to sound. As the blowing velocity is reduced below v, at those velocities exactly equal to the velocities defined by equation 11 which give the same edgetone frequency f but in the higher edgetone modes, we expect the pipe to sound again in its fundamental first harmonic mode at the exact first harmonic frequency f. At blowing velocities sufficiently near these exact velocities, the frequency sounded should be near the frequency f, the exact frequency depending upon the organ flue pipe Q-factor and how much the blowing velocity differs from the value of equation 11. This is exactly the pipe behavior described by Lord Rayleigh and summarized in Reference 2. Since the frequency is still the same, we expect the pipe's Q-factor to be the same for all these edgetone modes, and the variation with blowing velocity of the frequency sounded in these additional modes should be explained by the same theory presented in Reference 2.

As the pipe is blown with velocities increasingly higher than the velocity v giving the edgetone frequency f in mode 1, the situation is somewhat different. There are no additional edgetone modes to be excited that would result in the edgetone frequency f as the velocity is increased above v. Theoretically the phase condition on oscillations defined in Reference 2 can still be satisfied as the velocity goes to infinity, since the resonant air column of the flue pipe oscillator corrects for the phase shift introduced into the feedback loop, but the gain condition defined can't be satisfied since the gain around the feedback loop drops as the frequency excited departs from the exact resonant frequency of the air column, as shown by equation 3 of Reference 2. At some point the oscillation must cease. If the pipe is sufficiently overblown, that is, if the blowing velocity becomes high enough, the pipe's overtones begin to sound. Theoretically, for idealized conditions, the overtones of the pipe at exact resonance are exact integer multiples of the first harmonic or fundamental mode pipe frequency, but in practice because of end effects, these overtones will not be exact integer multiples of the frequency of the

fundamental mode, but will be very near such values. The theory I will present is easily modified to take account of these small differences. I will assume with small error that the overtones are exact integer multiples of the first harmonic so that the possible pipe resonant frequencies are given by the equation

```
f(m) = m*f(1)                                                (12)

m = 1, 2, 3, 4, ...                                          (13)
```

The parameter m is the mode number or harmonic number of the frequency produced.

```
f(m) = the frequency of the m-th harmonic of the pipe
and

f(1) = the frequency of the first harmonic or
fundamental mode.
```

Substituting f(m) into equation 11, we obtain

```
v(m, n) = G*m*f(1)/a(n)                                      (14)
```

NOTA BENE: Since the parameter G in equation 11 was defined for a turbulent plane jet and the transit time to the edge determined for this plane turbulent jet it is implicitly assumed in the following discussion of the flue pipe oscillator that the aperture and edge of the jet-edge system of the flue pipe are such that a plane jet impinging against a plane edge results. This means that we are considering a flue pipe of square cross section and not the perhaps more usual circular cross section.

All terms on the right side of equation 14 have been previously defined.

v(m,n) is thus the blowing velocity that we postulate as required to obtain the frequency m*f(1) with the edgetone mode n excited. Each pair of values (m,n) defines a possible mode of organ pipe operation. We expect the Q-factor of a mode will not depend upon n but will depend upon m since the frequency sounded does depend upon m but not upon n. The exact frequency sounded in each mode will depend upon the departure of the blowing velocity from the value defined by equation 14 for that mode, upon the Q-factor of the flue pipe for that mode, and upon the phase shift introduced into the feedback loop of the flue pipe oscillator, just as calculated by equation 4 of Reference 2. Velocities higher than v(m,n) give frequencies higher than m*f(1) and velocities lower give lower frequencies.

Whether or not the pipe will actually sound in mode (m,n) depends upon whether or not the loop gain condition defined in Reference 2 is satisfied in that mode, the phase condition should be automatically satisfied. For some values of blowing velocity, either the phase condition or loop gain condition or both will not be satisfied and thus regions of blowing velocity exist for which the pipe cannot sound. This predicted behavior is again exactly analogous to the operation of the ordinary electronic oscillator.

## Summary

The prediction of the present paper is that the organ flue pipe sounds at or near the frequencies m*f when the blowing velocity is at or near the blowing velocities v (m,n) predicted by equation 14. This prediction depends upon the postulated analogy between the ordinary electronic oscillator and the flue pipe oscillator being a valid analogy, which I believe is the case. There are not at present sufficient experimental data available to test the theory. An experiment such as the experiment of Lord Rayleigh's summarized in Reference 2, but extended in velocity variation and with the Q-factors of the pipe also being determined, would suffice to test the theory. Also the blowing velocity rather than blowing pressure given by Lord Rayleigh should ideally be the parameter measured. With present day technology, doing the experiment would be much simpler and considerably less difficult than it was for Lord Rayleigh. Equation 4 of Reference 2 predicts that a phase shift is introduced into the feedback loop of the flue pipe oscillator as the blowing velocity varies from the blowing velocity giving exact resonance of the pipe. This introduced phase shift is compensated by the opposite phase shift introduced by the action of the air column as the frequency sounded departs from the exact resonant frequency of the air column, so the phase condition required for oscillations to occur is satisfied. The purpose of the experiment would be to determine the value of this phase shift introduced into the feedback loop as calculated from equation 4 of Reference 2 as a function of the departure of the blowing velocity from the velocity giving exact resonance of the pipe in the mode (m,n). This would allow a great deal of information about the operation of the flue pipe oscillator to be deduced, and would allow the postulated relation between the flue pipe oscillator and the edgetone oscillator to be checked.

Like most theories of oscillator operation, this is a first order linear theory which cannot take account of non-linear effects, which usually determine which modes are excited, mode jumping, the exact points of oscillation onset and failure, the harmonic content of the tone produced, the amplitude of oscillation, and may affect the exact frequency achieved. All of these are important questions which the present theory does not attempt to answer. However, linear theories usually predict the general features of oscillator operation, operating conditions, and

oscillation frequencies quite accurately.

## References

(1) W. Blocker, The Theory of the Edgetone Oscillator, 1974; available on the Internet at http://www.nmol.com/users/wblocker/index.htm
(2) W. Blocker, On the Pitch of Organ Pipes, 1973; available on the Internet at http://www.nmol.com/users/wblocker/index.htm
(3) Lord Rayleigh, On the Pitch of Organ-Pipes, Phil. Mag. XIII, 340-347,(1882); and Lord Rayleigh's Scientific Papers 2, 95



Return to The Home Page of Wade Blocker. Page

Comments will be appreciated.
wblocker@nmol.com

# A THEORETICAL PREDICTION

## OF THE

# WEISENBERGER SCALING LAWS

## FOR THE

# ORGAN FLUE PIPE

October 29, 2000


## Wade Blocker

Comments will be appreciated
wblocker@nmol.com


## Abstract


The principles of a theory of the edgetone oscillator are applied to predict Richard J. Weisenberger's scaling laws for the organ flue pipe oscillator.


## Table of Contents



## Introduction

Edgetone oscillations have been explained in a previous paper (Ref. 1), and the variation in pitch of an organ flue pipe acoustic oscillator as the blowing pressure (or better, blowing velocity) is changed has been explained in a second paper (Ref.

2). Relying upon the close analogy between electronic and acoustic resonators, the application of principles from the theory of electronic oscillators allows numerical predictions to be made which are in very close agreement with the available experimental data for these acoustic oscillators. These papers give a diagram of the edgetone oscillator and define more fully most of the terms used in this paper. It is suggested that these papers be read before the present paper. The same theoretical principles underlying these papers are applied in this paper to predict the scaling laws discovered by Richard J. Weisenberger for the organ flue pipe oscillator.

## The Edgetone Oscillator

The column of air enclosed in a pipe or tube associated with such musical instruments as the pipe organ, flute, and recorder is not necessary for a tone to be produced. Only the jet-edge system consisting of the jet aperture and an edge separated by some distance X from the aperture is necessary. We will assume a plane jet having a thickness b at the jet aperture with an initial jet particle velocity V at the jet aperture. The jet is directed against the edge and oscillations of the jet occur producing a tone. A large number of modes of vibration of the jet in the gap X is possible with the frequency of the oscillation produced being a function of jet thickness b at the aperture, distance X between aperture and edge, initial jet particle velocity V at the aperture, and the particular mode of oscillation that is excited. Mode 1 (also called stage 1) of this edgetone oscillator is of particular importance to the pipe organ since the edgetone oscillator operating in mode 1 is the usual exiting agent for the organ flue pipe. Reference (1) gives a complete discussion of the theory of this edgetone oscillator.

## The Flue Pipe Oscillator

In the organ flue pipe and similar musical instruments such as the recorder and flute, a jet of air is blown through an aperture, traverses a distance X between the aperture and an edge and impinges against this edge. In these oscillators this jet-edge system is coupled to one end of a column of air enclosed in a long pipe of usually either square or circular cross section. This enclosed column of air is a resonant acoustic circuit. The organ flue pipe oscillator can be considered and analyzed (Ref. 2) as just an edgetone oscillator to whose feedback loop a resonant acoustic circuit has been added, which by virtue of its Q-factor can exert a very strong control over the frequency of oscillation. With appropriate dimensions of the jet-edge system, initial velocity of the jet of air, and length of the pipe, an oscillatory motion of the jet in the gap between the jet aperture and the edge results and pressure oscillations occur in the air column in the pipe. The result is a musical tone. The frequency of the tone is a function primarily of the length of the

pipe, which determines the resonant frequency of the acoustic resonator formed by the column of air in the pipe, but the initial jet velocity and the dimensions of the jet-edge system also affect the frequency but ordinarily to a much lesser extent. That is, the frequency is usually considered to be a function of initial jet particle velocity V at the jet aperture (or equivalently of the blowing pressure P), of the gap width X from aperture to edge, and the resonant frequency of the air column in the pipe, with this resonant frequency of the air column being of greatest importance. The frequency also depends upon the thickness b of the jet at the aperture, and upon the Q-factor of the resonant acoustic circuit formed by the air column. These latter dependencies are ignored in the usual discussion of the organ flue pipe oscillator. A discussion of the dependence of frequency upon all of these factors can be found in the other papers at this web site (References (2) and (3) of this paper).

## The Weisenberger Scaling Laws for the Organ Flue Pipe

During research into the properties of the organ flue pipe oscillator carried out in the late 1970s and early 1980s Richard Joseph Weisenberger discovered very important scaling laws that apply to the design and operation of the organ flue pipe oscillator. These scaling laws are set forth on his Web page (Ref. 4) entitled "Flue Pipe Acoustics" at the Internet address http://rjweisen.50megs.com/ with examples of their application. A search of the Web using the Google or other search engine with the search phrase "flue pipe acoustics" will turn up other examples of Weisenberger's research results and many other pages devoted to the organ flue pipe.

Weisenberger started with a flue pipe that produced the frequency F when the blowing pressure was PRS1 and the aperture to edge gap distance was X1. We will designate this as Case 1 and the numeral 1 affixed to PRS and X indicates that they are values for Case 1. In Case 1 the drive power necessary to run the compressor of the air supply was DRVPWR1, the flue pipe's acoustic output sound power was PWROUT1, and the efficiency with which drive power was converted to sound power was EFF1. Weisenberger then increased the aperture to edge distance to the value X2 and increased the blowing pressure to that value PRS2 for which the organ flue pipe operation was again satisfactory and which caused the pipe to sound with approximately the same frequency F. He found new values DRVPWR2, PWROUT2, and EFF2 for drive power, output sound power, and efficiency. We will designate this new set of conditions as Case 2. He then found the following relations (idealized from his experimental results) to exist between the parameters for Case 2 and those for Case 1:

```
(EFF2/EFF1) = (X2/X1)
```

^1 (1)

(PRS2/PRS1) = (X2/X1)
^2 (2)

(DRVPWR2/DRVPWR1) = (X2/X1)
^3 (3)

(PWROUT2/PWROUT1) = (X2/X1)
^4 (4)

The symbol ^ indicates exponentiation so that the quantities in parentheses are raised to the indicated powers. The indication of the exponent in the first relation is not necessary but is shown for symmetry with the other relations. We can apply Bernoulli's relation between blowing pressure and blowing velocity to the second of these relations and deduce a relation that is equivalent to that of equation 2.

(V2/V1) = (X2/
X1) (5)

where V2 and V1 are the blowing velocities at the jet aperture produced by the blowing pressures PRS2 and PRS1.

We will call the relations 1 through 5 the Weisenberger Scaling Laws for the Organ Flue Pipe.

As stated by Weisenberger, when the gap distance is doubled, to obtain the same frequency the blowing pressure must be quadrupled, that is, it must be increased by a factor 4. The compressor drive power required increases by a factor 8. The output sound power increases by a factor 16. And the efficiency of converting drive power to output power increases by a factor 2.

These are exceedingly important relations discovered by Weisenberger since they show the changes that must be made in the organ flue pipe to obtain any desired level of output power. Very importantly, these relations also show how to increase the efficiency of the oscillator.

The ratio X2/X1 cannot be increased indefinitely. Weisenberger in his papers indicates that the mouth area of the gap between the jet aperture and the edge against which the jet impinges should not exceed the cross sectional area of the organ pipe opening into which the jet blows. Both X1 and X2 should be such that

this restriction is satisfied.

## A Theoretical Derivation of the Weisenberger Scaling Laws

It has been shown previously (Ref. 1) that the frequencies f observed in the edgetone oscillator are predicted by the following theoretical equation, which has been verified by comparison with the available experimental data.

$$fT = a(n) \tag{6}$$

f is the frequency produced by the oscillator,

T is the transit time of a jet particle from the jet aperture to the edge, and

a(n) is defined by the theoretical sequence of numbers, derived in Reference 1,

$$a(n) = 0.500,\ 1.230,\ 2.239,\ 3.242,\ 4.244,\ 5.245,\ \ldots \tag{7}$$

for mode numbers n equal to 1, 2, 3, 4, 5, 6, ...

The parameter n is the mode number (also called stage number) of the edgetone oscillation mode that is occurring.

Equation 6 can be written as

$$f = a(n)/T \tag{8}$$

If T is the same fixed value for all modes then equation 8 predicts that the frequency f in each mode is different.

Equation 6 can also be written in the form

$$T = a(n)/f \tag{9}$$

where T is now the transit time required if we are to obtain the frequency f in mode n of the edgetone oscillator.

It was also shown in Ref. 1 that for a turbulent plane jet the transit time T(x)(note that the symbol X is being reserved for the value of x for the position of the edge)

of a jet particle from the jet aperture to the distance x from the aperture is given by

```
T(x) = T0*{[1 + (x/x0)]^(3/2) - 1}
(10)

T0 = (2/3)*[x0/V]
(11)

x0 = 5.75b
(12)

b = the aperture slit width, or equivalently the
thickness of the plane jet immediately out of the jet
aperture

V = the initial jet particle velocity at the jet
aperture
```

It will be noted from equations 10, 11, and 12 that when X is not appreciably greater than the thickness of the jet at the jet aperture the transit time T required for the jet particles to reach the edge at the distance X is approximately

```
T = X/V                                                (13)
```

This approximation to the transit time T(X) is not valid in the general case but is usually valid for stage one of the edgetone oscillator and apparently because of the constraints imposed on the organ flue pipes by practical considerations applies almost universally to the transit time of the jet particles from aperture to edge in the organ flue pipe oscillator. This approximation that the transit time T equals X/V for the organ flue pipe oscillator is a key element of the derivation below of the Weisenberger scaling laws.

We are postulating that ideally in the organ flue pipe oscillator the frequency of the organ flue pipe's jet-edge system considered as a edgetone oscillator should equal the frequency the organ flue pipe oscillator is designed to produce. (This is not a critical assumption since the Q-factor of the resonant acoustic column serves to lock the frequency of the organ flue pipe at or very close to the naturally resonant frequency of the pipe (see Ref. 2) so that very wide departures from ideal conditions are compensated for.) Therefore when the aperture to edge distance is increased from X1 for Case 1 to X2 for Case 2, equation 9 shows that the transit time T should be the same for both cases. Equation 13 then shows that the initial jet velocity V should be increased in the same proportion that X was increased.

Therefore we necessarily require that

$$(V2/V1) = (X2/X1) \qquad (14)$$

Equation 14 is the same as equation 5, and we have already seen that, by virtue of Bernoulli's equation, equation 5 is equivalent to equation 2, so we have predicted one of Weisenberger's scaling laws.

Next we note that the flow rate of mass in the jet has increased by the ratio (V2/V1), which because of equation 14 means that this flow rate of mass has increased by the ratio (X2/X1). The kinetic energy per unit mass has increased by the ratio (V2/V1)^2 or equivalently by the ratio (X2/X1)^2. Therefore, if we denote the kinetic energy flow rate by KE2 and KE1 in the two cases, we have the ratio

$$(KE2/KE1) = (X2/X1)^3 \qquad (15)$$

This energy must be provided by the compressor drive power so, if we assume that all efficiency factors in converting compressor drive power into kinetic energy of the jet are the same for both Case 2 and Case 1, equation 15 is equivalent to equation 3. Therefore we have predicted another of Weisenberger's scaling laws. But note that no assumption is involved in equation 15 so that equation 15 is more fundamental than equation 3, and should be somewhat more accurate.

In deriving the equations of Reference (1) for the edgetone oscillator it was necessary to assume that kinetic energy of the jet particles is converted to sound energy of the oscillations as the current of jet particles interacts with the acoustic oscillating pressure field of the oscillations in the space between the jet aperture and the edge. Equation 13 is equivalent to the approximation that the jet particles do not slow down in crossing the aperture to edge gap, so the efficiency of energy conversion is independent of position in the gap. We therefore expect the efficiency of energy conversion to increase proportionally with gap width X. Therefore we predict that the ratio of efficiency in Case 2 to the efficiency in Case 1 should be

$$(EFF2/EFF1) = (X2/X1) \qquad (16)$$

This equation is the same as equation 1 so we have predicted another of Weisenberger's scaling laws.

Combining equation 16 with equation 15, we predict the ratio of sound power in

Case 2 to the sound power in Case 1 will be

```
(PWROUT2/PWROUT1) = (X2/X1)^4
(17)
```

We have now predicted theoretically all of Weisenberger's experimentally established scaling laws for the organ flue pipe oscillator, using exactly the same theoretical principles that were used in establishing the results that are given in Reference 1 for the edgetone oscillator .

## Acknowledgements

The author wishes to express his thanks to Richard Weisenberger for his e-mail message of Thursday, October 26, 2000, calling attention to Weisenberger's "Flue Pipe Acoustics" website (Ref. 4) and to his experimentally established scaling laws for the organ flue pipe oscillator. This was the author's first acquaintance with these important quantitative facts about the behavior of organ pipe oscillators. The expectation immediately arose that the theoretical principles underlying the paper "The Theory of the Edgetone Oscillator" would apply here, and that it should be possible to predict theoretically the Weisenberger Scaling Laws.

## References

(1) W. Blocker, The Theory of the Edgetone Oscillator, 1974; available on the Internet at http://www.nmol.com/users/wblocker/index.htm
(2) W. Blocker, On the Pitch of Organ Pipes, 1973; available on the Internet at http://www.nmol.com/users/wblocker/index.htm
(3) Lord Rayleigh, On the Pitch of Organ-Pipes, Phil. Mag. XIII, 340-347,(1882); and Lord Rayleigh's Scientific Papers 2, 95
(4) R. J. Weisenberger, "Flue Pipe Acoustics" at the Internet address http://rjweisen.50megs.com/



To return to the Home Page, click on Home Page

Comments will be appreciated.
wblocker@nmol.com

# A Theoretical Model for the Edgetone Oscillator


## Wade Blocker

wblocker@nmol.com


March, 2004

This paper presents and rationalizes a simple model for the edgetone oscillator, the development of which led to the paper "The theory of the edgetone oscillator." That paper is available as the lead paper on the web site where this paper is found. It is recommended that that paper be read in order to understand every point discussed here. This paper is being presented to clarify and further rationalize the model on which I based "The theory of the edgetone oscillator."

The edgetone oscillator ceased to be of interest to mainstream physicists over sixty years ago which is probably the reason it has remained unexplained. It appears that everyone now working in the field starts with the misconception that the phase velocity of the jet wave in the oscillator is a small fraction of the jet particle velocity because that is what everybody keeps on saying, parroting each other, a misconception that these workers seem very reluctant to give up. But a review of the origins of this erroneous belief shows that it is not justified by the available data, and shows in fact that it is seriously wrong. Part of the problem is that they don't define very well just what they mean when they state their belief. This problem is discussed in my paper.

The author of "The theory of the edgetone oscillator" has brought it to the attention of a few authors of books or papers that discuss the edgetone oscillator. It is the opinion of this author that none of these present a correct view of the relation between the phase velocity of the jet wave and the velocity of the jet particles. They don't even seem to recognize that the jet particles slow down markedly as they cross the aperture to edge gap in this oscillator. This makes it impossible for them to develop a successful theory of the edgetone oscillator.

None of these has openly challenged the validity of the application in my paper of Schlichting's equation for the slowing of the jet particles to Brown's experimental data, from which every conclusion of mine about the edgetone oscillator can be shown to follow without the interposition of any further theoretical considerations. Nevertheless a simple theory can be presented that still requires Schlichting's equation to complete the application of the theory but predicts a relation between the phase velocity of the jet wave and the velocity of the jet particles that is made independently of Schlichting's equation, but which prediction the application of Schlichting's equation to Brown's data verifies. It also predicts that the edgetone oscillator is a transit time oscillator, which prediction the application of Schlichting's equation to Brown's data also verifies.

The authors don't like the theory I present for the jet wave in the gap and challenge my conclusions on that basis, ignoring the fact that my conclusions don't depend upon the theory of the jet wave that I present. The conclusions can be and were drawn from my theory of the jet wave, but they are independent of that theory. As I point out in the paper under discussion they can be drawn just as well on a purely empirical basis from Schlichting's equation applied to the available experimental data, particularly to that of Brown. To challenge my conclusions requires that either Schlichting's equation or Brown's data, or both, be shown to be incorrect. This has not been done and presumably cannot be done.

The critical assumption of my theory of the jet particle motion in the gap is that the oscillator is a push-pull oscillator. Everything follows from that assumption. I assume that the edgetone oscillator is a push-pull oscillator because there is mirror symmetry about the x-z plane through the aperture and the edge. In the common electronic oscillators mirror symmetry in the circuitry results in a push-pull oscillator, so I assumed that mirror symmetry results in the same thing for the edgetone oscillator. That means the driving forces on the jet particles are effectively 180 degrees out of phase on the two sides of the stream of particles. My equations for the jet wave follow in straightforward fashion from that assumption. I don't have to justify this assumption *a priori*, the assumption has instead the *a posteriore* justification that it leads to predicted results that are in agreement with the experimental facts. Actually this how all theories in physics have been established, they have been postulated first and then justified by the agreement of their predictions with the experimental facts. It is not even necessary that the initial assumptions be correct. It is just necessary that they incorporate enough of the truth that they lead to valid conclusions. Some theorists discussing the edgetone or the organ flue pipe, professional physicists even, seem to forget this fundamental fact at times.

You don't have to have a mental picture of how things work in physics, although

of course people are much more comfortable about things when they do have such a picture. The most successful physical theory in existence is quantum mechanics, and there is no one in the world who has a mental picture of how it works, no one can claim to understand it, although there are innumerable people who know how to apply it. The results that quantum mechanics predicts are in agreement with the experimental facts and therefore quantum mechanics is accepted as a correct theory. No exceptions to its predictions have yet been found. The equations I present for the edgetone oscillator suffice to predict all the basic experimental results of operating an edgetone oscillator and by the same standards that apply to quantum mechanics you would have to say that I have a correct theory. There are no competing theories that can predict the experimental results. This is the first theory developed that can predict Brown's and Carriere's experimental results for the edgetone oscillator.

The edgetone oscillator is still in the area of classical physics so a mental picture of its operation is surely possible, but it is not necessary. However for the edgetone oscillator a qualitative picure or model of its operation is easily developed and a theory based on this qualitative picture will give quantitative predictions that are in agreement with all the available experimental data.

If the 180 degree phase difference postulated above and in my paper "The theory of the edgetone oscillator" actually exists then the particles in the jet stream in alternate half cycles of the edgetone oscillation should be directed in alternating puffs into the upper and lower halfspaces, that is directed alternately above and below the plane of mirror symmetry. Such an alteration in particle motion should cause a corresponding shift in where the center of the pressure wave is. The center of pressure should be alternately above and below the plane of symmetry. That shift in position justifies the idealization, or simplification if you will, that there are pressure variations on the the two sides of the stream of jet particles (that is, above and below the jet stream in my presentation) that are 180 degrees out of phase. The assumed pressure difference gives rise to a predicted particle motion and this predicted particle motion gives rise to the assumed pressure difference on the two sides of the jet stream. That means that the feedback loop exists which every oscillator with a continuous output requires. This picture does not have to be demonstrated *a priori* to be correct although one hopes that it is and indeed it appears that it might be, its real justification is *a posteriore*, that it leads to predictions in agreement with the experimental facts. Such agreement is the justification for the use of the assumptions.

The predictions deduced from the model are in complete agreement with the experimental facts so the assumptions of the model are justified.

Return to The Home Page of Wade Blocker.

Comments will be appreciated.

wblocker@nmol.com